



THE TWENTY-FOURTH DUBROVNIK ECONOMIC CONFERENCE

Organized by the Croatian National Bank

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Unbundling Quantitative Easing: Taking a Cue from Treasury Auctions

Hotel "Grand Villa Argentina"

Dubrovnik

June 3 – 5, 2018

Draft version

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CROATIAN NATIONAL BANK

Unbundling Quantitative Easing: Taking a Cue from Treasury Auctions*

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March 14, 2018

Abstract

To understand the effects of large-scale asset purchase programs recently implemented by central banks, we study how markets absorb large demand shocks for risk-free debt. Using high-frequency identification, we exploit the structure of the primary market for U.S. Treasuries to isolate demand shocks. These shocks are sizable, leading to large movements in Treasury yields and impacting corporate borrowing rates. Informed by a preferred habitat model of the term structure, we test for “local” demand effects and find evidence consistent with theoretical predictions. Crucially, this local effect is strongest when financial markets are disrupted. Our estimates are consistent with the view that quantitative easing worked mainly via market segmentation, with a potentially limited role for other channels.

Keywords: quantitative easing, monetary policy, market segmentation, treasury auctions

JEL Classification: E52, E43, E44

*We thank Michael Bauer, Rhys Bidder, Michael Fleming, Ed Knotek, Matteo Maggiori, Eric Swanson, Michael Weber, and seminar participants at Berkeley, New York Fed, Cleveland Fed, San Francisco Fed, ASU, CMC, and Northwestern for comments on an earlier version of the paper. We are grateful to Maxime Sauzet for excellent research assistance.

1 Introduction

Demand for safe assets, and U.S. Treasuries in particular, plays a central role in the macro-financial landscape. To offset the negative effects of the recent financial crisis, central banks have implemented various large scale asset purchases, representing a sharp increase in demand for these assets. The most salient of these is the quantitative easing (QE) programs carried out by the Federal Reserve, which involved two trillion dollars of Treasury security purchases. Apart from the massive scale of these purchases, the Federal Reserve disproportionately bought long-term government debt, thus departing from the practice of having the distribution of its portfolio close to the distribution of outstanding debt (Figure 1).

While evaluating the program, Ben Bernanke, the chair of the Fed at the time, observed, “The problem with QE is it works in practice but it doesn’t work in theory.” Indeed, QE was successful in reducing short- and long-term interest rates, but the mechanism behind this reaction is still not well understood. For example, standard macro-financial models imply that the demand for assets such as Treasuries is determined solely by economic agents’ intertemporal consumption decisions, which does not capture the sources of demand shifts initiated by the Fed. Although the workhorse macroeconomic models cannot readily explain the workings of the QE, several explanations have been put forth. For instance, QE could be effective because it signaled to the markets that the Fed is serious about keeping short-term interest rates low for a long time (forward guidance). Or, perhaps the Fed exploited frictions (limited arbitrage and market segmentation) in the financial markets by purchasing securities in a particular segment. Finally, by buying assets on a massive scale, the Fed could signal a poor state of the economy which pushed interest rates down (“Delphic” effect; see [Campbell et al. \(2012\)](#) for more details).

Future deployment of unconventional tools such as QE requires policymakers to move beyond the “heat of the moment” policies, and hence a central question for policymaking and academic research is which of these theories is the key channel. But given the paucity of QE events, it has proven remarkably hard to provide clear empirical evidence for each theory, as well as to assess the relative contributions of the proposed channels. Indeed, many channels were likely active during QE rounds and the reactions to QE were observed in a particular state of the economy, which potentially confounds identification and interpretation.

The objective of this paper is to unbundle QE by focusing on one channel: market segmentation and *preferred habitat*, which posits that certain investors have preferences for specific maturities. Our approach is to identify shifts in *private* demand for Treasuries that mimic QE, but are independent of the other channels discussed above. The key mechanism through which market segmentation and preferred habitat forces operate is not the source of demand shifts, but rather how marginal investors in the market for

Treasury debt absorb these demand shocks. Therefore, the best way to isolate and study the preferred habitat channel of QE is to identify unexpected demand shifts that are unrelated to other channels of QE.

In order to construct demand shocks with these properties, we utilize the structure and timing of the primary market for Treasury securities. Similar to the empirical monetary policy literature (e.g., [Bernanke and Kuttner \(2005\)](#), [Gürkaynak et al. \(2007\)](#), [Gorodnichenko and Weber \(2016\)](#)), we look at high-frequency (intraday) changes in prices of Treasury futures in small windows around the close of Treasury auctions to identify unexpected shocks to demand for Treasuries. The key for identification is that all of the “supply” information (e.g. security characteristics such as the maturity, as well as the amount of newly offered and outstanding securities) is known and priced in by the market. For small enough windows around the close and release of the auction results, any price changes are reactions to information regarding the demand for the Treasury securities from the given auction. We interpret these price changes as demand shocks. Utilizing high-frequency changes in asset prices along with the timing of Treasury auctions in this manner allows us to rule out confounding factors and identify unexpected shifts in demand in a model-free way.

Treasury auctions have a number of properties that can help us understand the workings of QE. First, although the auctions are not as large as the QE rounds, the Treasury sells about \$150 billion in notes and bonds per month in recent years. Because the primary market for Treasuries is a convenient venue for investors who wish to purchase large amounts of government securities, the release of Treasury auction results can reveal potentially large shifts in demand for Treasuries. The surprise movements in the yields are reasonably large: a typical (one standard deviation) shock is equivalent to a yield change of roughly 2 basis points, which is much larger than similar changes on non-auction dates. For comparison, [Chodorow-Reich \(2014\)](#) estimates that the first round of the QE program in the U.S. cut Treasury rates (five-year maturity) by 9 basis points following the announcement from Chairman Bernanke on December 1, 2008.

Second, we document that demand shocks are driven by institutional investors such as foreign monetary authorities, investment funds, insurance companies and the like. Moreover, these shocks are not driven by changes in expectations about inflation, output, or other broad market conditions. Therefore, variation during Treasury auctions can help us to isolate the effect of idiosyncratic purchases in specific asset segments on the level and shape of the yield curve, which is difficult to achieve by examining only QE events.

Finally, in sharp contrast to QE events, Treasury auctions are frequent and information is available back to 1979. This gives us an opportunity for crisper inference and to study state-dependence in the effect of targeted purchases of assets (e.g., crisis vs. non-crisis states). Because QE events were both infrequent and confounded with a massive financial crisis, having a long time series is instrumental for understanding how QE-like programs

can work in normal times.

Importantly, because Treasury auctions for specific maturities are spread in time, we can identify changes in demand for government debt of specific maturities. As a result, we can trace how a shock in one part of the yield curve propagates to other parts of the yield curve. In this sense, we have natural experiments which can mimic targeted purchases of the Fed during QE programs. Hence, despite the apparent distance between QE programs and unexpected movements in private demand during regular Treasury auctions, this empirical strategy provides clean identification of demand shifts and allows us to map out the impact of these shocks.

Although we do not have a structural interpretation of unexpected changes in demand, we can still use shocks in demand for specific maturities of government debt to investigate how these shocks spread to other maturities. Specifically, we examine reactions across maturities through the lens of a formal preferred habitat theory of the term structure. Building on [Vayanos and Vila \(2009\)](#), we present a series of numerical simulations to provide qualitative predictions about how the location of the shock in maturity space affects the relative change in the term structure and how the reaction depends on the risk-bearing capacity of marginal investors. Informed by theory, we test these predictions using daily changes in spot rates for government debt in response to our measures of surprise movements in private demand at particular maturities. We find evidence consistent with our theoretical predictions.

Our results suggest that QE programs can be effective in influencing interest rates for debt at specific maturities when financial markets are disrupted. On the other hand, QE programs are less likely to be effective at this task in normal times when risk-bearing capacity of arbitrageurs is greater. In this case, demand shocks at a specific maturity likely move the whole yield curve rather than a specific segment, and the response may peak at a maturity other than the targeted maturity. Furthermore, if the Fed attempts to use purchases of debt with specific maturities to shift down the whole yield curve during a crisis, this exercise may be ineffective and the Fed should intervene at multiple maturities.

Furthermore, our results provide a quantitative sense of *how much* QE programs could influence interest rates through the preferred habitat channel. Specifically, using our regression estimates, we show that the amount of government debt purchases during the QE1 program should generate declines in yields similar to what was observed in the data. In other words, given the reaction of yields to surprise movements in *private* demand during Treasury auctions, we can account for most of the market reaction to QE1 announcements. This result is consistent with the view that QE worked mainly via market segmentation and preferred habit, and that the *net* effect of other channels was small.

Our study contributes to several strands of previous research. First, we provide new evidence to the literature examining theoretically (e.g., [Vayanos and Vila \(2009\)](#)) and

empirically (e.g., [Greenwood and Vayanos \(2014\)](#), [Krishnamurthy and Vissing-Jorgensen \(2012\)](#), [Hamilton and Wu \(2012\)](#)) determinants of demand for government debt. In particular, we add to the literature departing from the “expectations hypothesis” (e.g., [Kuttner \(2006\)](#)) of the term structure of interest rates, and provide evidence for alternative explanations such as limited arbitrage and market segmentation. Our findings are complementary to [Lou et al. \(2013\)](#) and [Fleming and Liu \(2016\)](#) who also utilize Treasury auctions to explore how *supply* shocks interact with these forces. Second, we contribute to the rapidly growing literature studying the effects of QE programs in the U.S. and other countries (see [Martin and Milas \(2012\)](#) for a survey) and in particular the literature studying how market segmentation interacts with QE programs (e.g. [D’Amico and King \(2013\)](#)). While most of these studies focus on market movements around QE announcements (e.g., [Krishnamurthy and Vissing-Jorgensen \(2012\)](#), [Chodorow-Reich \(2014\)](#)), we instead focus on market movements around Treasury auctions that can also give us an opportunity to investigate market reactions to unexpected changes in demand for government debt not only in crisis but also in normal times. Third, our paper is methodologically related to earlier studies (e.g., [Kuttner \(2001\)](#), [Bernanke and Kuttner \(2005\)](#)) utilizing high-frequency data to construct surprise movements in policy. Although we do not measure unexpected movements in policy, we construct shocks in private demand that inform us about how markets can react to changes in policy.

2 Data and Institutional Details

In this section we describe the primary sources of our data and present basic statistics. First, we describe the U.S. Treasury auctions for U.S. government notes and bonds (coupon-bearing nominal securities). Second, we describe the details of futures contracts for these Treasury securities.

2.1 Primary Market for Treasury Securities

The Treasury sells newly issued securities to the public on a regular basis through auctions. Currently, 2-, 3-, 5- and 7-year notes are auctioned monthly. 10-year notes and 30-year bonds are auctioned in February, May, August and November with reopenings in the other 8 months. The frequency of auctions evolved over time. For example, 30-year bonds were not issued between 1999 and 2006 and were issued only twice a year between 1993 and 1999.

There are two types of bids: noncompetitive and competitive. Noncompetitive bidders agree to accept the terms settled at the auction, and are typically limited to \$5 million per bidder. Competitive bidders submit the amount they would like to purchase and the price (the interest rate) at which they would like to make the purchase. For each competitive

bidder, the submitted amount cannot be greater than 35% of the amount offered at the auction.

Auction participants include primary dealers, other non-primary brokers and dealers, investment funds (for example, pension, hedge, mutual), insurance companies, depository institutions, foreign and international entities (governmental and private), the Federal Reserve (System Open Market Account), and individuals. These participants are classified into three groups. The first group is Primary Dealers (brokers and banks) that trade on their accounts with the Federal Reserve Bank of New York. This group typically buys the largest share of auctioned debt and is required to participate in every Treasury auction. The second group is Direct Bidders: non-primary dealers submitting bids for their own proprietary accounts. The third group is Indirect Bidders who submit competitive bids via a direct submitter, including Foreign and International Monetary Authorities placing bids through the Federal Reserve Bank of New York.¹

Additionally, the Treasury divides investors into the following classes: Investment Funds (mutual funds, money market funds, hedge funds, money managers, and investment advisors); Pension and Retirement Funds and Insurance Companies (pension and retirement funds, state and local pension funds, life insurance companies, casualty and liability insurance companies, and other insurance companies); Depository Institutions (banks, savings and loan associations, credit unions, and commercial bank investment accounts); Individuals (individuals, partnerships, personal trusts, estates, non-profit and tax-exempt organizations, and foundations); Dealers and Brokers (primary dealers, other commercial bank dealer departments, and other non-bank dealers and brokers); Foreign and International (private foreign entities, non-private foreign entities placing tenders external of the Federal Reserve Bank of New York (FRBNY), and official foreign entities placing tenders through FRBNY); Federal Reserve System (the Federal Reserve Banks System Open Market Account (SOMA)); Other (represents the residual from categories not specified in investor class descriptions above). [Fleming \(2007\)](#) describes in greater detail the breakdown by types and class of bidders.

As detailed in [Figure 2](#), there are four stages of a Treasury auction:²

1. *Announcement*: A few days before an auction, the Treasury releases all the pertinent information regarding the upcoming auction. An announcement includes security information (maturity, CUSIP identifier, schedule of coupon payments, etc.) as well as

¹Additionally the Federal Reserve System purchases securities for its System Open Market Account (SOMA). Starting in 1997, the SOMA amount was changed from being listed within the announced offering amount to being additions to the announced offering amount. That is, if the Treasury auctions \$15 billion in bonds and the Federal Reserve would like to purchase \$1 billion in the auction, the Treasury issues \$16 billion in bonds. This change was made so that the Treasury would be able to provide better information to the market about the amount of securities actually available for sale to the public.

²See [Driessen \(2016\)](#) for details on the design of Treasury auctions. [Garbade \(2007\)](#) provides historical details regarding the manner in which the Treasury has conducted auctions.

the amount offered, the bidding closing times, which class of bidders can participate, and other information describing the rules of the auction.

Figure 3 presents a typical announcement. At this auction, the Treasury offers \$16 billion in 30-year bonds. This is a new auction (that is, the Treasury does not reopen a previous auction) with the maximum award (that is, maximum allocation to a bidder) of \$5.6 billion.

2. *Bidding*: After the announcement, individuals and institutions may submit bids up until the closing times of the auction. The announcement in Figure 3 stipulated that non-competitive bids should be submitted by 12:00 p.m., while the deadline for competitive bids is 1:00 p.m.
3. *Results*: Most Treasury note and bond auctions close at 1:00 p.m. Competitive bids are accepted in ascending order (in terms of yields) after the auction closes until the quantity meets the amount offered minus the amount of non-competitive bids. All bidders receive the same yield as the highest accepted bid.³ Once the auction closes and the winning bids are determined, the information regarding the results is released immediately. Besides the winning yield, the Treasury announces various aggregate statistics regarding the bidding. Beginning in the early 2000s, auction results are released within minutes of the close of the auction (see [Garbade and Ingber \(2005\)](#)).

Figure 4 presents a typical announcement about auction results, which corresponds to the auction announcement presented in Figure 3. The demand (tendered) for the security was \$33.3 billion, most of the bids came from primary dealers (\$23.7 billion), \$489.9 million was bought by the Federal Reserve (SOMA), and a relatively low amount was bought via non-competitive bids (\$14.8 million). The “bid-to-cover”, the ratio of all bids received to all bids accepted, was $\$33.3/\$16.0=2.08$. The interest rate, corresponding to the winning yield, was set at 3.75 percent per year.

4. *Issuance*: A few days after the close of an auction, the Treasury delivers the securities and charges the winning bidders for payment of the security. At this point the winning bidders can hold the security to maturity and receive coupon payments, or sell the security on the secondary market.

Data from the announcements and results of every auction since late 1979 are available from [TreasuryDirect.gov](#). Data regarding amounts accepted and tendered by bidder type (Primary Dealer, Direct, and Indirect) are available starting in 2003. Additionally, the Treasury provides information regarding allotment by investor class (Investment Funds, Individuals, etc) starting in 2000.

³Between 1970 and 1992, Treasury did not charge a uniform price. Instead, allocation of bonds was made at the individual yields stipulated by the bidders.

2.2 Treasury Futures

We use Treasury futures prices in order to construct market-based measures of demand surprises occurring during Treasury auctions. Treasury futures are standardized contracts that obligate the seller to deliver a valid Treasury security to the buyer at a later date. Futures contracts for 30-year Treasury bonds were introduced in 1977, followed later by 10-year, 5-year, and 2-year Treasury note futures. Treasury futures currently trade on the Chicago Mercantile Exchange (CME), and intraday tick-level data are available starting in 1995. The market for Treasury futures is deep: the average daily volume of trade in 2012 was more than 2 million contracts with more than \$100 billion of notional value.

The futures contracts close in March, June, September, and December. We focus on the “closest” contract, i.e. the contract that closes within 1-3 months as these are by far the most traded. For example, in February we use the March expiry, while in March we use the June expiry. Although contracts that close in a given month can still be traded, the volume of trades is substantially lower.

Note that futures are not tied to any specific bond issue (CUSIP). Each futures contract allows for a range of deliverable Treasury securities. 2-year futures contracts allow for delivery of Treasury notes with remaining maturity between 1-year 9-months to 2 years; 5-year futures allow for remaining maturity between 4-year 2-months to 5-year 3-months; 10-year futures allow for remaining maturity between 6-year 6-months to 10-years; and 30-year futures allow for delivery of Treasury bonds with remaining maturity of at least 15 years.⁴ In principle any permissible Treasury security can be delivered into a futures contract, but as explained in [Lauszweski et al. \(2014\)](#) in practice a so-called “cheapest to deliver” (CTD) security emerges for a given futures contract. Although which Treasury security is used for payment can vary over time, this variation happens at relatively low frequencies (weekly or monthly) and therefore our analysis at high frequencies should not be materially affected by this peculiarity of Treasury futures contracts.

Because futures cannot be matched to a specific CUSIP, we link a given auction to Treasury futures using the maturity offered in the auction. For example, if the Treasury auctions 7-year notes, we use the 10-year futures contract which allows delivery of securities maturing in at least 6.5 years and no more than 10 years. While this linking introduces a mismatch in terms of maturities, the difference between the maturity of matched futures contracts and the maturity of the auctioned government debt is relatively small.

⁴The 30-year futures contract is also known as the “classic T-Bond” future. This contract originally allowed for delivery of bonds with remaining maturity between 15 years and 30 years. In 2009, the CME Group introduced “Ultra T-Bond Futures” which uses Treasury bonds with remaining maturity of at least 25 years but no more than 30 years, and changed the range of deliverable maturities to the classic contract to bonds with remaining maturity between 15 years and 25 years. While the “Ultra” futures contract provides a better match for long maturities, the time series for the contract is relatively short and the volume of trades is small relative to the other longer-running futures. For these reasons we use the “Classic T-Bond Futures” to ensure consistency over time.

We use Treasury futures prices for a number of reasons. First and foremost, Treasury futures provide a natural market-based measure of unexpected shifts in Treasury prices. Further, Treasury futures trade on a standardized exchange rather than over the counter. Another option would be to use “when-issued” prices of the Treasury securities being auctioned. Although this option has the benefit of matching perfectly the security being auctioned, the downside is that the when-issued market systematically trades at lower yields than the winning yield at the auction (see e.g. [Fleming and Liu \(2016\)](#)).⁵

2.3 Summary Statistics

In our analysis we focus on Treasury note and bond auctions. We exclude inflation protected securities (TIPS), floating rate notes (FRNs), cash management bills (CMBs), and callable bonds (the last of which was issued in 1984), because these securities have different structural arrangements than simple coupon-bearing nominal securities. We also exclude Treasury bills (zero-coupon securities with maturity one year or less) because the QE programs mainly bought long-term nominal U.S. government debt.⁶ Further, Treasury futures contracts exist for 2-year, 5-year, 10-year, and 30-year nominal Treasury notes and bonds, but not for shorter term bills.

Figure 5 plots the number of note and bond auctions per year in our sample, broken up by term length. The number of auctions is relatively stable throughout the 1980s to mid 1990s. In the face of declining government debt, the number of auctions temporarily fell in the late 1990s and early 2000s, which also coincides with the termination of new issuances of 30-year bonds. After the Great Recession, the number of auctions increased significantly.

Table 1 presents summary statistics for note and bond auctions between 1979 and 2015, and the subsample 1995-2015 for which we have intraday Treasury futures prices. Since 1995, a typical offering is about \$20 billion which generates more than \$50 billion in demand so that the bid-to-cover ratio is approximately 2.6. The largest source of demand for Treasuries is primary dealers (their bid-to-cover ratio is ≈ 2) but other types of bidders also account a large fraction of auction offerings. Primary dealers purchase approximately 60 percent of auctioned Treasuries with the rest split equally between investment funds and foreign buyers.

There is considerable variation in the offered amounts (standard deviation is $\approx \$9$ billion) as well as the level and composition of demand (standard deviation for the bid-to-cover ratio is ≈ 0.5 and the standard deviation of bid-to-cover ratio for primary dealers is 0.35). In our sample of Treasury note and bond auctions, the median maturity is 5 years.

⁵In 2015, the U.S. Department of Justice launched a probe to investigate whether various financial companies (most of them are primary dealers of U.S. Treasury securities) participated in a conspiracy to manipulate the “when issued” market for Treasuries.

⁶For example, between November 3, 2010 and June 29, 2011 (QE2), the Fed bought \$750 billion in Treasury securities, of which TIPS purchases were only approximately \$26 billion.

The winning yield (“high yield”) is on average close to 3.2 percent per year with standard deviation of 1.9 percentage points. The distribution of submitted bids tends to be fairly compressed: the high-median yield spread is approximately 3 basis points with standard deviation of 2 basis points. However, on some occasions the spread can be as high as 10 basis points.⁷

3 Quantifying Demand Shocks

In this section, we describe how we measure the surprise movements in prices of Treasury futures around Treasury auctions and document properties of these surprises. Our key assumption is that within small enough windows around the close and release of Treasury auction results, shifts in the prices of Treasury futures reflect unexpected changes in market beliefs about the demand for Treasuries with a specific maturity. Indeed, since the Treasury announces an offered amount well before an auction happens thus fixing supply, between the announcement and close of the auction futures prices should move only in response to changes in demand conditions. By focusing our analysis on a narrow window around the close time of an auction, we likely isolate variation only due to unexpected shifts in demand for this specific auction. As a result, we can identify a demand shock for a specific maturity and then use this shock to trace the reaction of Treasury futures prices for the given maturity and for other maturities as well as reactions for other parts of the financial market.

3.1 Shock Construction

Let $P_{t,pre}^{(m)}$, $P_{t,post}^{(m)}$ be the futures prices before and after the close of the auction on date t with maturity $m = 2, 5, 10, 30$. We measure the surprise movements in the futures prices as:

$$D_t^{(m)} = \log P_{t,post}^{(m)} - \log P_{t,pre}^{(m)}. \quad (1)$$

These surprises are computed for all maturities at date t irrespective of what maturity is being auctioned on date t . In other words, we compute $D_t^{(2Y)}$ (surprise movement in the 2-year Treasury futures) not only for auctions that offer 2-year government notes but also for auctions that offer Treasuries with other maturities.

For all auctions, $P_{t,pre}$ is the last price observed 30 minutes before the close of the auction. For auctions taking place between 1995 and 1999, $P_{t,post}$ is the first price observed 1.5 hours after the close of the auction; after 2000, we use the first price observed 30 minutes after the close of the auction. The Treasury began releasing results much faster

⁷Between 1999-2015 when the data is available, the Fed purchased Treasuries through SOMA in approximately two thirds of auctions; when doing so they purchased an average of \$2.3 billion (standard deviation of \$2 billion).

in the early 2000s, but in the 1990s auction results frequently took over an hour after the close of the auction to be released. Unlike the close of the auction, the time at which the results are released is not reported by the Treasury. However, wire reports from Bloomberg allow for an upper bound on the release time. Note that we use small symmetric windows around the events to eliminate predictable movements in prices identified in [Lou et al. \(2013\)](#) and [Fleming and Liu \(2016\)](#). Indeed, [Fleming and Liu \(2016\)](#) show that these predictable movements extend to the hours before and after the auction, but near the close of the auction and release of the results the price movements are reactions to the surprises regarding the demand observed at the auction. Hence, the use of small intraday windows is key to identifying unanticipated demand shocks.

Figure 6 plots the time series of our constructed shock measures, with summary statistics presented in Table 2. Panel A of Table 2 reports summary statistics for $D_t^{(m)}$ shocks during auction dates (our main sample). The mean values of the shocks are close to zero suggesting that surprises are not systematic and do not contain predictable movements. The standard deviation of $D_t^{(m)}$ increases in maturity m . To verify that these shocks are not spurious we also report (Panel B of Table 2) movements in futures prices on non-auction days (for days without auctions, the same “pre” and “post” windows are used as auctions in the same year). In all cases, the variance of the shocks on auction dates is larger than on non-auction dates. This pattern is consistent with auction results indeed influencing futures prices.

The table also reports moments for the zero lower bound (ZLB) and pre-ZLB periods. The variability of surprises for short maturities is considerably smaller during the ZLB period (December 2008 to the end of our sample) than outside the ZLB period (1995 to December 2008). For longer maturities, the volatility is similar for ZLB and pre-ZLB periods. However, these statistics mask important heterogeneity. As seen in Figure 6, during the Great Recession the volatility of surprises was elevated but then we observe strong compression for short maturities since the economy enters recovery. This finding is consistent with [Swanson and Williams \(2014\)](#) documenting that while the Fed’s policies during the Great Recession compressed fluctuations of short-term rates, the behavior of long-term rates is still relatively normal.

Note that the shocks are in terms of futures (log) prices. Although futures contracts do not have a natural definition of yield, an approximate yield can be computed using the Treasury securities delivered at the end of the contract. Using this approximation, a one standard deviation change in the log price of each contract is equivalent to a 2.0 to 2.5 basis point change in yield for each contract.⁸

Additionally, Table 2 documents that price changes of Treasury futures strongly comove across maturities, with the strongest correlations between adjacent maturities. For

⁸For details on how to convert between Treasury futures prices and the yield on the corresponding “cheapest-to-deliver” Treasury security, see [Lauszweski et al. \(2014\)](#).

example, on auction dates the correlation between $D_t^{(10Y)}$ and $D_t^{(30Y)}$ is 0.922 while the correlation between $D_t^{(2Y)}$ and $D_t^{(30Y)}$ is 0.672. Note that the correlations are generally stronger between short ($D_t^{(2Y)}$) and longer maturities during the non-ZLB period than during the ZLB period. At the same time, the comovement of $D_t^{(5Y)}$, $D_t^{(10Y)}$ and $D_t^{(30Y)}$ does not appear to be materially influenced by the binding ZLB. These correlations suggest that shocks to a given segment of the maturity spectrum generally affect not only prices of that particular segment but also prices in other parts of the spectrum, but there is heterogeneity across time in the strength of the correlation. This is a key result, which we explore in detail in Section 4.

3.2 Narrative Evidence

To provide a better understanding of what forces are behind these surprise movements, we plot the 30-year Treasury futures price during two 30-year Treasury bond auctions (Figure 7). The first is from an auction on August 11, 2011. Futures prices were relatively stable in the lead up to the close of the auction, but after the close and release of the auction results prices dropped sharply and immediately. The Financial Times [wrote](#):

“An auction of 30-year US Treasury bonds saw weak demand...bidders such as pension funds, insurers and foreign governments shied away. ‘There’s not too many ways you can slice this one, it was a very poorly bid auction.’”

The second is from December 9, 2010. This auction was a reopening of previously issued 30-year bonds from the month prior. Once again, the futures prices are relatively stable in the lead up to the close of the auction. After the auction closes and results are released, prices immediately spiked up. The Financial Times [wrote](#):

“Large domestic financial institutions and foreign central banks were big buyers at an auction of 30-year US Treasury bonds on Thursday. ‘Investors weren’t messing around...You don’t get the opportunity to buy large amounts of paper outside the auctions and ‘real money’ were aggressive buyers.’”

We interpret the two example auctions as follows. Before the auction closes, the market information set consists of all the supply information, both for outstanding securities as well as the amount on offer for the current 30-year auction. The 30-year futures prices reflect beliefs about the expected path of short-term interest rates, inflation expectations, and demand for long-term Treasury securities. After the auction closes and the results are released, the only update to the information set is the news regarding the bidding that took place in the auction, which solely reflects demand for Treasury debt. The change in the 30-year futures price reflects this unexpected shift in beliefs about Treasury demand. The contemporaneous articles in the financial press further suggest that the important driver of the demand shifts arise from foreign and domestic institutional investors. The

last example also highlights why auctions can have important elements of price discovery: when investors have to purchase large amounts of Treasuries to meet their needs, they may prefer to use auctions rather than attempting to make substantial transactions on the secondary market. As a result, auctions reveal new information about demand.⁹

3.3 Demand Determinants

Our assumption is that $D_t^{(m)}$ captures unexpected shifts in the demand for Treasuries. We further hypothesize that these shocks are particularly driven by demand shifts arising from institutional investors. Figure 7 and the corresponding reporting in the financial press provided some narrative evidence in this direction. However, $D_t^{(m)}$ is a market-based measure and hence is an equilibrium response to the underlying shifts in demand. Because the mapping from shifts in demand to changes in futures prices may be complex, it is important to establish that the market interpretation of changes in demand is actually related to observable movements in demand.

One of the most commonly reported statistics in the financial press is the bid-to-cover ratio. It is a natural measure of the demand at a given auction (the higher is the bid-to-cover ratio, the higher is demand). The bin scatter plot in Figure 8 shows that the bid-to-cover ratio (after controlling for its four own lags) is a strong predictor of our measure of demand shocks. Table 3 presents more formal evidence by regressing our shocks on measures of demand reported at the auction:

$$D_t^{(m)} = \alpha^{(m)} + \beta^{(m)} X_t^{(m)} + \varepsilon_t^{(m)} \quad (2)$$

This specification is estimated separately for auctions corresponding to the Treasury futures maturity groups in columns (1)-(4). For example, column (1) restricts the sample to include only auctions of 2-year notes and column (2) restricts the sample to include only auctions of notes with (2,5] year maturity. Column (5) reports results when we pool across maturities and impose that $\beta^{(m)}$ is the same across maturities m . To facilitate the comparison of the results, we standardize $D_t^{(m)}$ in these regressions to have zero mean and unit variance.

Panel A of Table 3 estimates equation (2) using the current bid-to-cover ratio as well as four lags of the bid-to-cover ratio (winsorized at the 1% level). These coefficients may be interpreted as the reaction of $D_t^{(m)}$ surprises to an innovation in the bid-to-cover ratio, and correspond to the slopes of the regression lines in Figure 8. The results show that the bid-to-cover ratio is positively associated with $D_t^{(m)}$ and the effect of an increase in the bid-to-cover ratio is statistically and economically significant. For example, a one

⁹We could not find any reference in the press about monetary policy (or leaked information about future monetary policy) being a source of unexpected movements. Consistent with this observation, we do not find any statistical power of surprise movements in Treasury futures around Treasury auctions to predict future monetary policy.

standard deviation (0.5) increase in the bid-to-cover ratio (after controlling for its own four lags) in a Treasury auction for 30-year bonds raises the price of the 30-year Treasury futures by $2.119 \times 0.5 = 1.06$ standard deviations (this corresponds to a 0.26 log point increase in the price of the Treasury futures or an approximate change of 2.5 basis points in the yield).¹⁰

Panel B repeats the regressions from Panel A, but explicitly decomposes the bid-to-cover ratio into “expected” and “surprise” components. For these regressions, we first estimate a univariate AR(4) model of the bid-to-cover ratio, separately for each maturity group. We then construct the fitted (expected) and residual (surprise) values of the bid-to-cover ratio, and regress $D_t^{(m)}$ on these expected and surprise components. We find that the variation in our demand shocks is determined by the surprise component of the bid-to-cover ratio, and is unaffected by expected movements in the bid-to-cover ratio.

In order to assess sensitivity of futures prices to changes in demand by bidder type, Panel C reports estimates of equation (2) using the bid-to-cover ratio of Indirect Bidders, Direct Bidders, and Primary Dealers. The sensitivity of surprises D to unexpected demand of indirect bidders increases with maturity. For example, a unit increase in the bid-to-cover ratio for indirect bidders raises the price of 2-year Treasury futures by 2.7 standard deviations and the price of 30-year Treasury futures by 8.5 standard deviations. For direct bidders, the sensitivity is highest for short maturities. The sensitivity to changes in the bid-to-cover ratio coming from primary dealers for 2- and 5-year Treasury futures is smaller than the sensitivity for 10-year Treasury futures and greater than the sensitivity for 30-year Treasury futures. When we pool across maturities, demand of direct and especially indirect bidders generates *ceteris paribus* more variation in futures prices than demand of primary dealers.

Panel D uses additional investor allotment data from the Treasury to break down the amount accepted by types of bidders: Investment Funds, Foreign, Dealers, and Miscellaneous. Since the fractions by group add up to one, we set Dealers as the leave-out category. The estimated coefficients suggest that as the fraction accepted for investment funds and foreign buyers increases, $D_t^{(m)}$ increases too. The coefficients for the Miscellaneous category are generally smaller and less robust.

These results indicate that, indeed, a key determinant of $D_t^{(m)}$ surprises is movements in demand conditions as proxied by the bid-to-cover ratio. Furthermore, we observe that the demand from institutional investors is important in accounting for variation in $D_t^{(m)}$. In subsequent analyses, we will use this strong relationship to instrument $D_t^{(m)}$ with unexpected movements in the bid-to-cover ratio so that for our estimates we exploit variation due to changes in demand rather than variation due to fluctuations in market conditions.

¹⁰We found that controlling for other variables (e.g., policy uncertainty constructed in [Baker et al. \(2016\)](#)) in equation (2) does not materially change our estimates.

3.4 Comovement Across Markets

We now turn to analyzing how our demand shocks for Treasuries propagate across other financial markets. Given the relatively high degree of correlation across our demand shocks, in the following analysis we will find it useful to compress $D_t^{(m)}$ into a single summary statistics: the first principal component of $D_t^{(m)}$. This time series captures the general movement of the yield curve in response to demand shocks for government debt with various maturities. The first principal component explains 88 percent of variation in our shock measures. We denote the first principal component by D_t , which has zero mean and unit variance.

We measure the impact of demand shocks on other asset prices by running simple bivariate regressions:

$$y_t = \gamma + \phi D_t + u_t \quad (3)$$

where y_t is the change in the price or yield of some asset on auction date t . Where available, we use intraday changes within the same time window as our shocks D_t . However we also examine changes at the daily frequency, partly due to data limitations but also because daily changes may pick up responses in other asset markets that don't occur immediately. A strong correlation between D_t and y_t signals either that D_t and y_t have a common determinant (e.g., changes in inflation expectations alter the behavior of bids in Treasury auctions and change prices of inflation swaps) or that y_t is a channel of propagation for D_t shocks (e.g., unexpected prices in an auction result in repricing of Treasuries in the secondary market). To preserve space, we focus on the OLS estimates of equation (3) and report very similar instrumental variable estimates (using unexpected changes in the bid-to-cover ratio as instrumental variables) in Appendix Table B1.

Panel A of Table 4 reports results for debt markets. The dependent variable in the first two rows are the intraday change in the Exchange Traded Funds (ETF) "TLT" and "SHY", which track Barclays Capital U.S. 20+ Year and 1-3 Year Treasury Bond indices, respectively. The third row of the panel is the intraday change in the ETF "LQD", which tracks the iBoxx Liquid Investment Grade Index. The coefficient should be interpreted as the impact in log points of a one standard deviation change in D_t . In all cases we observe a strong reaction to the Treasury demand shock, accounting for more than 50 percent of variation observed in these ETF prices during the short windows around the close and release of the Treasury auction results.

The final row reports the results for the daily change in corporate bond yields, as measured by Moody's Aaa corporate yields. Consistent with the intraday results, the negative coefficient implies that an increase in the price of Treasury futures (which means that the yield on Treasuries falls) is associated with a decrease in the Aaa bond yields. Specifically, a one standard deviation shock to D_t decreases the Aaa bond rate by 2.3 basis points. However, using daily rather than intraday changes as the dependent variable leads

to a decline in R^2 s, which underscore the benefits of using intraday data.

As expected, yields in the secondary market react strongly to the demand shock. Furthermore, this reaction is persistent in spite of the fact that our shocks are constructed from intraday movements. Figure 9 plots the contemporaneous reaction of 10-year Treasury spot rates (top panel) and the Aaa corporate bond yields (bottom panel) to our shocks D_t , as well as the reactions up to 60 days in the future. The reaction remains strongly statistically significant nearly 1 month later, while the point estimate is remarkably stable even 2 months later.

Panel B of Table 4 reports results for equities. Rows 1 and 2 report the results for the intraday change in ETFs tracking the S&P 500 and the Russell 2000 indices. Rows 3 and 4 are for the daily changes in these indices. Although the estimated slope is generally negative, the quantitative significance of shocks on equities is small: these shocks account for a tiny share of variation in equities.

Panel C of Table 4 presents results for inflation expectations and commodities. Rows 1 and 2 report the results for the daily change in inflation expectations implied by inflation swaps at the 10-year and 2-year horizon. We observe that demand shocks for Treasuries do not generate significant movements in inflation expectations. To explore the robustness of this finding, we examine price reactions of two additional assets which are often used to hedge against inflation. The dependent variable in row 3 is the intraday change in the ETF “GLD,” which tracks the price of Gold Bullion. Row 4 reports results for the daily change in the S&P Total Commodity Index. For neither of these variables do we find a significant correlation with D_t . To further explore sensitivity of inflation expectations to demand shocks D_t , we plot reactions of inflation swap rates at all available maturities in Appendix Figure B1. We find that the change in the inflation expectation “yield” curve exhibits little reaction to D_t . We find a similar lack of sensitivity of inflation expectations when we use specific $D_t^{(m)}$ instead of summary series D_t . These results suggest that the demand shocks we measure at auctions are not driven by some underlying change in inflation expectations. Moreover, these shifts in demand do not propagate to changes in inflation expectations. This result is intuitive, as changes in demand of institutional investors or foreign monetary authorities are unlikely to generate future fluctuations in the rate of U.S. inflation.

Panel D of Table 4 reports results for various bond spreads and credit default swaps. The first row reports results for the daily change in the Moody’s Baa-Aaa corporate yield spread. Rows 2 and 3 use daily changes in two CDS indices from Credit Market Analysis (CMA) that track the automotive industry (a highly cyclical industry) and banks (a proxy for the financial sector). These three measures proxy for expectations about future output and market conditions. We find that surprise movements D_t have no tangible effect on these measures, consistent with the view that D_t shocks do not capture superior information of Treasury auction bidders about future recessions and the like. In row 4, we

document that D_t shocks are not associated with VIX (a measure of market perceptions about future volatility). Hence, it is unlikely that there is a common force that moves D_t and volatility or that D_t shocks propagate via volatility. Finally, we find (row 5) that D_t shocks are not significantly related to the 3-month LIBOR-Overnight Index Swap (OIS) spread (a measure of short-term liquidity risk) so that liquidity fluctuations are unlikely channels or determinants of D_t . In short, as with the case of inflation expectations, these null results suggest that our demand shocks are not being driven by changes in expectations regarding output, liquidity, default risk, or volatility

The results of Tables 3 and 4 allow for some broad observations. First, given our high-frequency approach and the structure of Treasury auctions, we know that our constructed shocks are only driven by new information regarding the demand side of the market. Second, these shifts are largely driven by shifts in the demand that arises from institutional investors. Third, as expected these demand shocks from the primary market propagate not only to the secondary market, but also to the corporate debt market. Finally, these demand shifts are not driven by some underlying shift in macroeconomic expectations (flight to quality, inflation expectations, etc.) that may move demand for Treasuries at all maturities. But it still remains the case that a variety of factors can generate movements in $D_t^{(m)}$ and one should not interpret $D_t^{(m)}$ as structural shocks.¹¹ Despite this limitation, the properties of $D_t^{(m)}$ shocks allow us to study how unexpected demand interventions at specific maturities propagate to other maturities.

4 Channels of Treasury Demand Shocks

Although demand shocks $D_t^{(m)}$ strongly comove with one another, the responses are not uniform across maturities. To see the heterogeneity in reactions, we use daily changes in zero-coupon spot rates as constructed in [Gürkaynak et al. \(2007\)](#), which provide more granularity to the analysis (recall that we have only four maturities in Treasury futures contracts while the yield curve utilizes information for many more maturities but is available only at the daily frequency). Figure 10 plots “responses” of changes in the yield curve for auctions on two dates. On the first date (August 11, 2011), there was unexpectedly weak demand (as measured by changes in futures prices) during an auction of 30-year Treasuries. We observe that, although the whole yield curve shifted up, the strongest reaction was at long maturities. On the second date (February 6, 2007), there was an auction of 3-year government notes, and demand during this auction was unexpectedly

¹¹To highlight this caveat, we plot sensitivities for select asset prices estimated over rolling windows in Figure B2. The sensitivity of LQD prices, Aaa interest rates, and Baa-Aaa spread is relatively stable over time. On the other hand, the sensitivity of S&P500 flips sign from positive in the late 1990s to generally negative since the early 2000s, which is consistent with [Campbell et al. \(2014\)](#). Although we do not have long-time series of inflation expectations (or assets used for hedging against inflation), we observe that during the Great Recession in the U.S., inflation expectations and D_t moved in opposite directions while in normal time these two series are approximately uncorrelated.

strong. The whole yield curve shifted down, but the strongest reaction was at the 8-year maturity. These two cases illustrate that the “propagation” of demand shocks across maturities does not amount to simple upward or downward shifts.

This raises the question: to what extent do Treasury demand shocks have local effects? In other words, does the location of the demand shock in maturity space matter? And are the impacts state-dependent? The two auctions in Figure 10 provide suggestive evidence that the location can in fact matter. In order to better characterize the impact of these demand shocks, we now examine the impact on the term structure of Treasury rates through the lens of the *preferred habitat* model of investor demand. The key idea is the existence of “clientèle” investors who have idiosyncratic demand for Treasuries of specific maturities. The other side of the market are risk-averse arbitrageurs, who smooth out these demand shocks. Using a version of the model from Vayanos and Vila (2009), we create qualitative predictions of what happens to the term structure when hit with demand shocks to various parts of the maturity space during different economic regimes.

4.1 Preferred Habitat – Numerical Exercise

In our numerical exercises, we consider a “three-factor” version of the Vayanos and Vila (2009) model consisting of the instantaneous rate, and two demand factors that are otherwise equivalent, but are located in the “short” and “long” ends of the maturity space. We then solve the model and study the impact of each demand shock. A key element of the model is the level of risk aversion of the arbitrageurs, hence we study the reactions as risk aversion increases from very low to very high. We consider the case where the “short” shock is concentrated at the 3-year maturity, while the “long” shock is at 20 years (corresponding to the average length of short-term and long-term auctions in our empirical section). See Appendix A for details regarding the model and parameterization.

Figure 11 shows the change of the term structure in response to the short (top panel) and long (bottom panel) demand factors, as the risk aversion of arbitrageurs increases from low (lighter lines) to high (darker lines). In the case of low risk aversion, the impact is very similar: rates fall across the entire term structure, but the impact peaks at the short end of the yield curve, then drops off as the maturity increases. The only difference between the impacts of the short and long shocks is that the long shock has a larger impact; but the shape of the response is nearly identical.

However, as risk aversion increases, the responses become quite different depending on the location of the demand shock. A shock to the short demand factor sharply decreases short-term rates; but this impact dies off quickly and even turns slightly positive at the long end of the term structure. On the other hand, the long demand factor decreases both short and longer term rates, but the impact is much stronger in the long end of the term structure.

These results confirm that some of the findings of [Vayanos and Vila \(2009\)](#) for the limiting cases of no risk aversion and infinite risk aversion also hold for intermediate cases of risk aversion. As they explain, the intuition for these results is as follows: when arbitrageurs are perfectly risk-neutral, demand shocks have no impact as the expected path of the instantaneous rate is the only determinant of the term structure. As arbitrageurs become somewhat risk averse, shocks to the instantaneous rate continue to be much more influential than demand shocks. But now arbitrageurs are concerned about instantaneous rate risk. Every bond is sensitive to instantaneous rate risk, and the market price of this source of risk is determined in equilibrium by the portfolio allocations of arbitrageurs. Because demand shocks from preferred habitat investors cause changes in these portfolios, even in cases of very low risk aversion these demand shocks do affect the term structure by altering the price of instantaneous rate risk.

What is the impact of the location of the demand shock? Consider an increase in preferred habitat demand. Regardless of the location of the shift, this causes arbitrageurs to sell bonds, reducing their exposure to instantaneous rate risk. Hence they require lower expected returns to hold bonds, pushing down rates. The location of the demand shock determines which bonds arbitrageurs sell; since these bonds have different sensitivity to instantaneous rate risk, this determines the magnitude of the overall reduction in rates. But regardless of the location of the demand shift, the bonds that respond the most are those most sensitive to instantaneous rate risk, which depends only on the stochastic properties of the instantaneous rate and demand shocks. In other words, when arbitrageurs have low risk aversion, the relative impact of short and long demand shocks to the term structure is roughly the same; only the overall size of the impact is affected by the location of the demand shock. In our calibration, for low values of risk aversion this leads to the peak impact occurring around $m \approx 4$. This could be lower or higher with different parameterizations, but remains largely independent of the location of the shock in maturity space.

As arbitrageur risk aversion increases, demand shocks become more prominent as additional sources of risk. Arbitrageurs try to limit their exposure to these sources of risk, leading to less propagation from the location of the demand shock to other parts of the term structure. This implies that the term structure response is more localized to each demand shock as arbitrageurs choose not to integrate bond markets across maturities. For example, following a demand shock for short-term bonds, all else equal arbitrageurs would like to buy longer term bonds to hedge the risk arising from the short demand shock, leading to upward pressure on prices (and downward pressure on spot rates). But this changes their exposure to the long demand shock as well, and this countervailing force leads arbitrageurs to sell sufficiently long-term bonds. As seen in the top panel of [Figure 11](#), when risk aversion is sufficiently high this countervailing force can become strong enough to lead to an increase in rates for very long-term bonds.

In summary, this illustrative exercise indicates several qualitative predictions. When risk aversion is low, the impact of an increase in demand for either short-term or long-term debt causes a decrease in rates everywhere. Moreover, while the magnitude may differ, the response to both demand factors are similarly shaped, peaking at intermediate maturities and declining for very long-term maturities. Conversely, when risk aversion is high, demand factors have a stronger local component: increases in demand for short-term debt will have a maximal impact on shorter-term maturities, while long-term shocks will peak at long-term maturities. Additionally, the response of long-term (short-term) rates to long (short) demand shocks increases as risk aversion increases, respectively. Finally, although the magnitude of responses is more ambiguous, when risk aversion is high we expect the response of short-term rates to short demand shocks will be larger than to long demand shocks; and vice versa for responses of long-term rates.

4.2 Empirical Results

Comparing the theoretical results from Figure 11 with Figure 10 suggest that, at least during the auction in Panel A, the preferred habitat model with relatively high risk aversion does a good job explaining the response of the term structure; Panel B is more ambiguous. We now take a more rigorous approach to testing the predictions of our numerical exercise.

A key variable is a measure of risk aversion of arbitrageurs. We proxy this using the measure of financial crises in the United States from Romer and Romer (2017).¹² The crisis index is a continuous measure derived from narrative sources to identify periods of financial distress (higher values correspond to periods of more extreme financial crisis). Besides identifying financial distress during the recent financial crisis, the measure also identifies periods of distress in 1986, the early 1990s, and 1998 (Figure B3 in the Appendix).

In order to measure the impact of demand shocks on the entire term structure, we estimate the following regression equations

$$\Delta R_t^{(m)} = C_t \left(\alpha^{(m,H)} + \beta^{(m,H)} D_t^{(m')} \right) + (1 - C_t) \left(\alpha^{(m,L)} + \beta^{(m,L)} D_t^{(m')} \right) + \varepsilon_t^{(m)} \quad (4)$$

for each maturity $m = 1, \dots, 30$. $\Delta R_t^{(m)}$ is the daily change in the zero-coupon spot rate for the given maturity as measured by Gürkaynak et al. (2007). C_t is an indicator

¹²He and Krishnamurthy (2013), Kyle and Xiong (2001) and others show how risk aversion can be endogenously higher in times of crises. Note that in contrast to other popular measures of financial stress (e.g., the Federal Reserve Board staff's Financial Stress Index), the Romer-Romer index does not use yields on Treasuries (outcome variables in our exercise) to identify stress/non-stress periods. Additionally, rather than using a narrative measure of financial distress, we used a market-based proxy for arbitrageur risk aversion. We define high risk aversion periods as those in which the "intermediary capital ratio" described in He et al. (2016) is low (Appendix Figure B4), and find similar results.

variable that is equal to 1 when the [Romer and Romer \(2017\)](#) measure of financial crisis is non-zero. The coefficients $\beta^{(m,L)}$ capture the impact of demand factor $D_t^{(m')}$ at maturity m' (our normalized intraday futures price shocks) during periods of low risk aversion; similarly $\beta^{(m,H)}$ capture the impact during periods of high risk aversion. While our shock measure is constructed at a higher intraday frequency, in order to capture the full extent of how markets absorb these shocks we prefer to use these daily estimates of the yield curve. To the extent that shocks are absorbed completely within smaller windows than a day, using daily changes as an outcome variable simply adds noise to our estimates, but shouldn't result in any bias.

A straightforward way in which to test the predictions of the preferred habitat model is to estimate equation (4) in two separate subsamples: *i*) days with short auctions; *ii*) days with long auctions. In our baseline regression we break up auctions into 2-7 years and 10-30 years. We choose the 10-year cutoff for long vs. short rather than 30-year in order to have a more balanced sample; the results are robust to choosing different cutoffs.¹³ Breaking up the auctions in this manner allows us to more closely pinpoint the location of the demand shock in the maturity space, and ties closely with the numerical exercise above.

For our measure of demand shocks $D_t^{(m')}$ on the right-hand-side of equation (4), in our baseline results we take the same approach above and match each auction with the (normalized) futures surprises of closest maturity (e.g. for 5-year auctions, use the 5-year futures surprise). The β coefficients should be interpreted as the response of spot rates for maturity m to a one standard deviation demand shock at maturity m' on the day when maturity m' is auctioned.

Figure 12 plots the low and high coefficients from the two subsamples (Appendix Figure B5 plots p-values testing for equality of the coefficients). During periods of low risk aversion, the impact of short and long demand shocks on the term structure closely mirror one another. Both shocks decrease spot rates across the entire term structure, and are hump-shaped. But when risk aversion is high, the short and long demand shocks have differential impacts. Both shocks exhibit stronger local effects. For the long shock, the impact is no longer hump-shaped as the impact continues to remain large as the maturity increases. The magnitude is also considerably larger than the corresponding responses during periods of low risk aversion. The impact of the short demand shock peaks at intermediate rates and then begins declining; further, the magnitude is larger than the corresponding response in periods of low risk aversion. Finally, when risk aversion is high, the response of short-term rates to short demand shocks is larger than their response to long demand shocks; and vice versa for the response of long-term rates.¹⁴

¹³Consistent with our exercise in Section 4.1, the average maturities of “short” and “long” auctions are 3 and 20 years respectively.

¹⁴A downside of using the yield curve data from [Gürkaynak et al. \(2007\)](#) is that idiosyncratic changes along portions of the yield curve may be smoothed out. To address this concern, we repeat our empirical

These results confirm the key predictions of our numerical exercise: during periods of low risk aversion, short and long demand shocks have relatively similar impacts; and these impacts peak at short to intermediate maturities. During periods of high risk aversion, the impacts are more localized: the impact of short demand shocks peak at short maturities, while the impact of long demand shocks peaks at the long end of the term structure. This local effect is particularly strong for long-term demand shocks. We also find that the peak responses for both short and long demand shocks are larger during periods of high risk aversion than during periods of low risk aversion.

As discussed above, the shocks $D_t^{(m')}$ are equilibrium reactions of market prices to changes in demand for Treasuries. As a result, these reactions may depend on market conditions (specifically on whether financial markets are disrupted) so that measured responses combine reactions to changes in demand and to how futures prices react to changes in demand. To isolate the effect of changes in demand for Treasuries, we employ unexpected changes in the bid-to-cover ratios for Treasury auctions as instruments for $D_t^{(m')}$ in equation (4). That is, we instrument $C_t \times D_t^{(m')}$ and $(1 - C_t) \times D_t^{(m')}$ with $C_t \times b_t^s$ and $(1 - C_t) \times b_t^s$, where b_t^s is the surprise movement in the bid-to-cover ratio for a Treasury auction at date t (as in Panel B of Table 3). This approach permits state-dependent mappings from demand shocks to futures prices. Consistent with our earlier results, we find that the unexpected changes in the bid-to-cover ratios are strong instruments for D_t (first-stage F-statistics above 10). The IV estimates of estimated reactions of spot rates (Figure 13) are similar to the OLS estimates (if anything, the responses during periods of financial distress are larger in magnitude, although confidence bands are wider) thus reassuring that our shocks D_t are good measures of the underlying demand shifts and hence the OLS estimates are capturing the response of the yield curve to these demand shifts.

As a robustness check, rather than use demand shocks as identified by intraday Treasury futures changes, we can also use the surprise component of the bid-to-cover ratio directly as a proxy of demand shocks from the auctions themselves. We re-estimate equation (4) using b_t^s in place of our demand shocks. Although the surprise component of the bid-to-cover ratio is not as clean a measure of demand shocks, this allows us to check the robustness of our results, as well as to expand the sample period to 1979-2015. Appendix Figure B7 plots the results using the same sample 1995-2015 as in Figure 12, while Appendix Figure B9 uses the entire sample 1979-2015 (p-values are in Appendix Figures B8 and B10, respectively). As expected, the standard errors are a bit wider, but the qualitative responses are generally similar.

exercise but use security-level yield changes in the secondary market for Treasuries. In place of the [Gürkaynak et al. \(2007\)](#) rate changes, we use the daily changes in the yields for Treasury notes and bonds in the secondary market from CRSP. We can no longer trace out changes to zero-coupon rates at all points along the maturity space, but by grouping together Treasuries of similar maturities we can test for local demand shocks more directly. Appendix Table B2 confirms the findings from Figure 12.

We also tried a number of additional empirical exercises, and find our results are robust to a variety of different specifications, including different cutoffs for short-term and long-term auctions and using different subsamples.¹⁵

5 Implications for QE

The responses of the yield curve to unexpected movements in demand during Treasury auctions offer several lessons for how one should understand the workings of QE programs implemented by the Fed and other central banks. For example, if the Fed is trying to decrease long-term Treasury rates relative to shorter-term rates, our results suggest that QE policies that directly purchase long-term Treasuries should be highly effective during financial crises. But if the Fed is trying to move the entire term structure of interest rates, during periods of high financial distress the Fed will have to be active in purchasing Treasuries throughout the yield curve. Thus, programs in spirit of “Operation Twist” may be an option because a central bank actively intervenes in multiple segments of the yield curve during a crisis.

As we move away from the most recent crisis, there have been discussions (see [Blinder et al. \(2016\)](#)) about whether central banks will continue to use unconventional policies in the future. Our results suggest that the impact of QE-style policies during non-crisis periods will likely differ greatly from those observed during the crisis. To the extent risk aversion is low and debt markets are more integrated, QE-type programs that attempt to move long-term rates relative to short-term rates may fail. During “normal” times of low risk aversion, the overall response of interest rates is less tied to the location of the shifts in demand. While we still expect targeted purchases of long-term Treasury debt from the Fed to reduce long-term rates, the largest declines may be for shorter term maturities that are not directly purchased by the Fed.

Interestingly, our results suggest that the Fed may have a menu of options in terms of where it can intervene in the maturity space to hit the yield at a target maturity. For example, suppose the Fed wishes to decrease 30-year Treasury rates by purchasing \$30 billion of notes and bonds. What is the benefit to targeting purchases in the longer

¹⁵Dropping auctions that occurred during the weeks of QE announcements leads to nearly identical results. In a handful of cases, auctions occur on the same days as FOMC announcements. Although our intraday windows do not overlap with the FOMC announcements, we also re-did our regressions dropping these dates, which leaves our results unchanged.

We additionally highlight one more robustness specification which more closely matches our numerical exercise. We take the first two principal components of our intraday shocks, D_t^ℓ and D_t^s , rotated such that D_t^s is uncorrelated with $D_t^{(30Y)}$ and normalized to have zero mean and unit variance. The first two principal components explain 97 percent of variation in our shocks. For long-term auctions the shock is D_t^ℓ ; similarly short-term auctions use D_t^s . In this way we have two distinct “short” and “long” demand factors, which more closely matches our numerical exercise. Appendix Figure B11 plots these results (p-values in Appendix Figure B12), and finds very similar results as the baseline specification. The only difference is the response of long-term rates to short demand shocks falls more closely to zero.

end of the term structure? If the economy is not in a crisis and financial markets are healthy, the answer is not much. Given the size of a typical Treasury note auction in recent years, these \$30 billion purchases represent a unit increase in the bid-to-cover. Our estimates imply that if the purchases were for short-term securities (2-7 years), we would expect to see an increase in our normalized demand shocks of 1.43 (standard error 0.19) and an ensuing decrease in secondary market 30-year rates of -1.84 basis points (0.36). If instead these purchases were of long-term securities (10-30 years), we would expect to see an increase in our normalized demand shocks of 1.90 (0.22) and a corresponding decrease in secondary market 30-year rates of -3.03 basis points (0.48). However, if these purchases took place in a period of financial turmoil, the purchases in the long end of the term structure relative to the short end become more effective. In this case, short-term security purchases lead to an increase in our normalized demand shocks of 1.39 (0.31) and a decrease in secondary market 30-year rates of -2.78 basis points (0.75). But long-term purchases lead to increase in our normalized demand shocks of 2.26 (0.38) and an ensuing decrease in secondary market 30-year rates of -7.22 basis points (0.86).

We can also use our results to assess what fraction of the market response to QE1 can be explained directly by shifts in demand for Treasury debt arising from the Fed.¹⁶ To summarize the timeline of the Fed's actions, there were five announcements during QE1, four of which mentioned purchasing long-term Treasury securities. November 25, 2008: the Fed announced purchases of \$100 billion in GSE debt and \$500 billion in MBS. December 1, 2008: Chairman Bernanke stated that the Fed could purchase long-term Treasuries. December 16: the FOMC announced possible purchases of long-term Treasuries. January 28, 2009: the FOMC announced it is ready to expand agency debt and MBS purchases, and to begin purchasing long-term Treasuries. March 18, 2009: the FOMC announced it will purchase \$300 billion in long-term Treasuries, along with an additional \$750 billion in agency MBS and \$100 billion in agency debt.

Using small intraday windows around the time of the four announcements which mentioned Treasury purchases, [Chodorow-Reich \(2014\)](#) estimates the 5-year Treasury rate reacted by -9.2, -16.8, 3.1, and -22.8 basis points, respectively. For the same dates but using larger 2-day windows to account for the possibility of slow responses due to liquidity effects, [Krishnamurthy and Vissing-Jorgensen \(2011\)](#) estimates the announcements moved the 5-year Treasury rate by -28, -15, 28, and -26 basis points respectively; additionally, they find the 5-year rate moved by -23 basis points after the initial November 25 announcement.¹⁷ This gives a range of cumulative decline of between 45 and 74 basis points. Note that, because QE1 set the stage for subsequent QE programs, this decline could combine the promise to purchase \$300 billion in Treasuries in QE1 with the possibil-

¹⁶We focus on QE1 since the surprise component of QE2 and QE3 was likely smaller than that of QE1.

¹⁷[Chodorow-Reich \(2014\)](#) drops the November 25, 2008, announcement because it occurred after trading hours. In addition, the positive response to the January 28, 2009, announcement seems to be because markets were expecting a concrete statement about purchases.

ity of additional rounds of quantitative easing that would entail buying more government debt. In other words, the 45-74 basis point decline could overstate the response of the markets relative to a response one could have observed in the case when the Fed credibly committed to spend only \$300 billion to purchase government bonds during the entirety of *all* its quantitative easing programs.

With this caveat in mind, we can carry out a back-of-the-envelope calculation to assess how much of the response of yields is due purely to the shift in demand for Treasuries from the Fed. The large majority of the Fed Treasury purchases during QE1 were concentrated in the 2-7 year range, and the magnitude of purchases would correspond to a ten-fold increase in the bid-to-cover ratio observed during auctions over the same period. During this period of financial distress, our estimates imply that we should expect a shock of this size to decrease 5-year secondary market spot rates by 44 basis points (95% confidence interval of 29–59 basis points). Our estimate is close to the estimates from [D’Amico and King \(2013\)](#), reporting that Treasury purchases during QE1 reduced yields by about 30 basis points.

Although this exercise represents a very large out-of-sample forecast for our data, it shows that the actual market reaction to QE1 announcements is consistent with the predictions of a preferred habitat model and the behavior of the market in response to observed shifts in *private* demand for Treasuries. Since the mechanism for the market segmentation channel is the same regardless of the source of demand shifts (recall that market segmentation is about how private arbitrageurs absorb these shifts rather than the source of the demand shocks), this finding implies that the *net* effect of other channels of QE (e.g., inflation expectations, forward guidance, signaling) could be smaller than thought before. Consistent with this observation, [Krishnamurthy and Vissing-Jorgensen \(2012\)](#) document that there was little movement in 5-year inflation expectations in response to QE1 announcements.

6 Concluding Remarks

Quantitative easing (QE) was a massive policy experiment which likely influenced the economy via multiple channels. To understand how QE worked, we need to unbundle these channels so that future policy can be designed to maximize the effectiveness of QE-like tools in crisis and non-crisis times. In this paper, we focus on the “preferred habitat” channel, which posits that, because of market segmentation and limited arbitrage, interest rates for a given maturity range may be influenced by targeted buying or selling of assets within this range. We utilize Treasury auctions of government debt to identify Treasury demand shocks arising from changes in institutional investor demand to study how shocks in one maturity segment propagate to other segments, and how this propagation is affected by the condition of financial markets.

While these shocks do not have structural interpretation, they provide us with variation that is not related to some prominent theories of how QE works (inflation expectations, forward guidance, signaling) and instead allow us to focus attention on the role of preferred habitat mechanisms. Crucially, these mechanisms are dependent on how private agents in the market for Treasury debt absorb these demand shocks, regardless of the source of these shocks. Therefore, we can use this variation to examine whether preferred habitat theory can rationalize responses of interest rates to unexpected changes in demand for government debt with specific maturities during regular Treasury auctions and, by extension, QE rounds.

We find a strong local component of demand shocks (i.e., with some oversimplification, purchases of assets in a particular segment move prices more strongly in that segment), but the local concentration is decreasing in risk-bearing capacity. That is, local effects are stronger when markets are segmented (e.g. due to a crisis) than when markets are integrated. The magnitude of the responses during Treasury auctions is large enough to account for a large part of interest rate movements in response to QE announcements, consistent with the view that QE programs worked mainly via market segmentation. Our analysis suggests that QE can be an effective policy tool in crises, but will be less powerful in moving specific segments of the debt market in normal times. Finally, the net contribution of other hypothesized channels of QE propagation may be quantitatively less important than thought before.

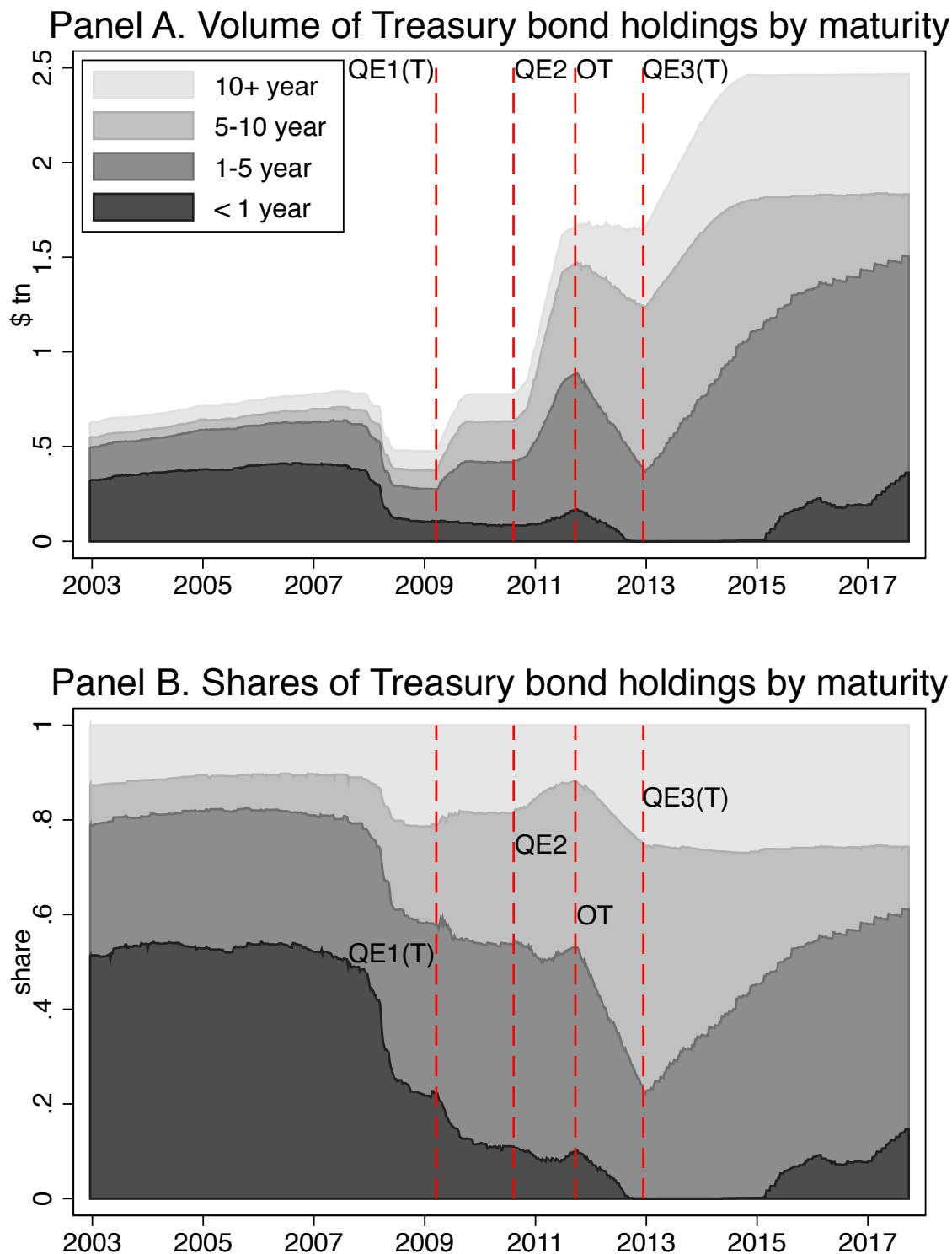
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Figure 1: Volume and composition of SOMA's holdings of U.S. Government Debt



Notes: QE1(T) denotes time when the Fed announced its decision to buy U.S. Treasuries as a part of the first round of quantitative easing. QE2 denotes the announcement of the second round of quantitative easing. QE3(T) denotes the time when the Fed announced its purchases of U.S. Treasuries as a part of the third round of quantitative easing. OT denotes the announcement of “Operation Twist”. Source: FRED database.

Figure 2: Auction Timing

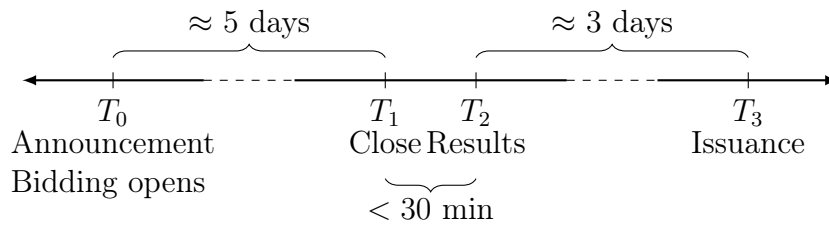


Figure 3: Example of an Auction Announcement

August 03, 2011

202-504-3550

TREASURY OFFERING ANNOUNCEMENT ¹

Term and Type of Security	30-Year Bond
Offering Amount	\$16,000,000,000
Currently Outstanding	\$0
CUSIP Number	912810QS0
Auction Date	August 11, 2011
Original Issue Date	August 15, 2011
Issue Date	August 15, 2011
Maturity Date	August 15, 2041
Dated Date	August 15, 2011
Series	Bonds of August 2041
Yield	Determined at Auction
Interest Rate	Determined at Auction
Interest Payment Dates	February 15 and August 15
Accrued Interest from 08/15/2011 to 08/15/2011	None
Premium or Discount	Determined at Auction
Minimum Amount Required for STRIPS	\$100
Corpus CUSIP Number	912803DT7
Additional TINT(s) Due Date(s) and CUSIP Number(s)	August 15, 2041 912834KP2
Maximum Award	\$5,600,000,000
Maximum Recognized Bid at a Single Yield	\$5,600,000,000
NLP Reporting Threshold	\$5,600,000,000
NLP Exclusion Amount	\$0
Minimum Bid Amount and Multiples	\$100
Competitive Bid Yield Increments ²	0.001%
Maximum Noncompetitive Award	\$5,000,000
Eligible for Holding in Treasury Direct Systems	Yes
Eligible for Holding in Legacy Treasury Direct	No
Estimated Amount of Maturing Coupon Securities Held by the Public	\$24,430,000,000
Maturing Date	August 15, 2011
SOMA Holdings Maturing	\$2,205,000,000
SOMA Amounts Included in Offering Amount	No
FIMA Amounts Included in Offering Amount ³	Yes
Noncompetitive Closing Time	12:00 Noon ET
Competitive Closing Time	1:00 p.m. ET

Figure 4: Example of an Auction Result Announcement

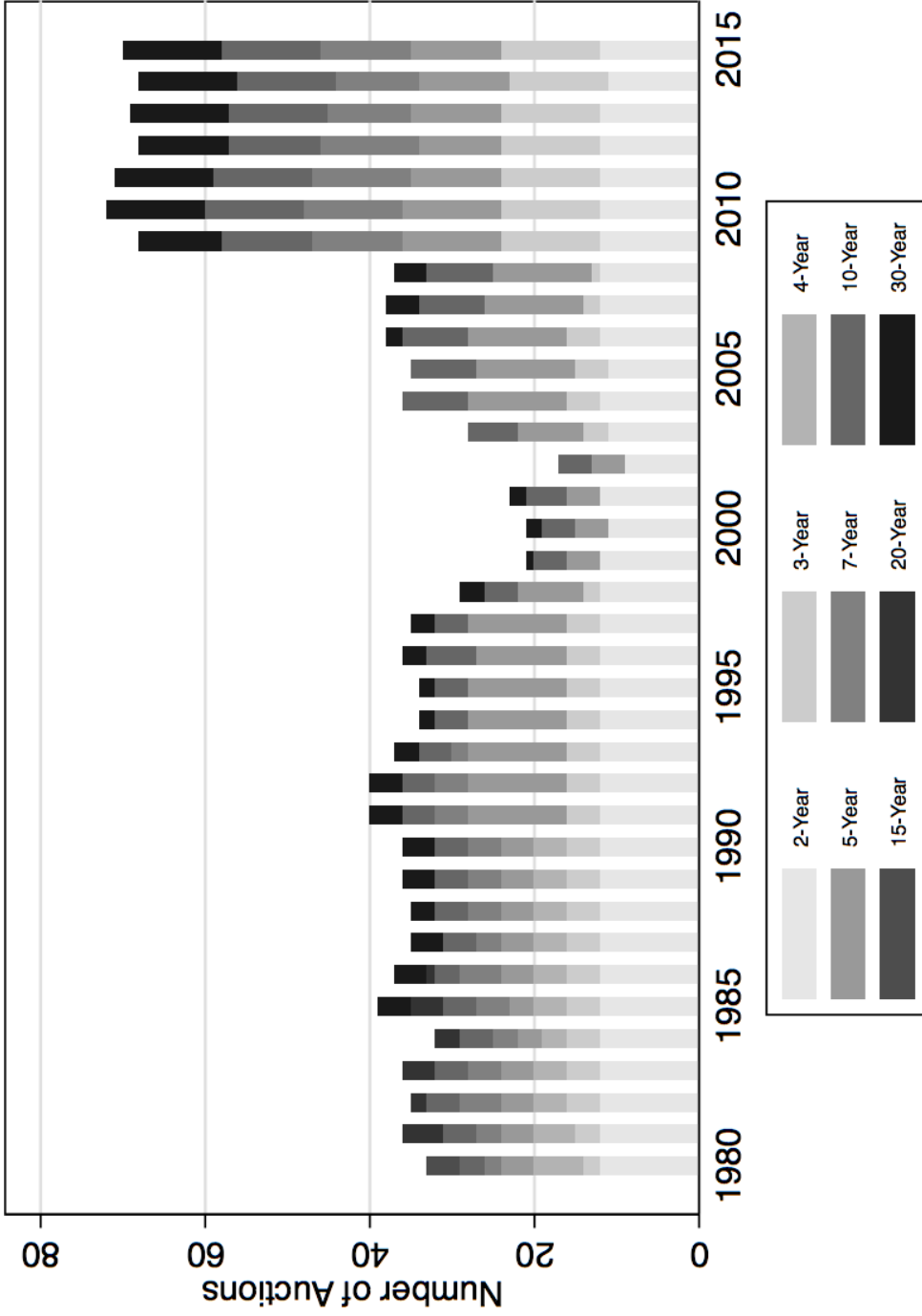
August 11, 2011

202-504-3550

TREASURY AUCTION RESULTS

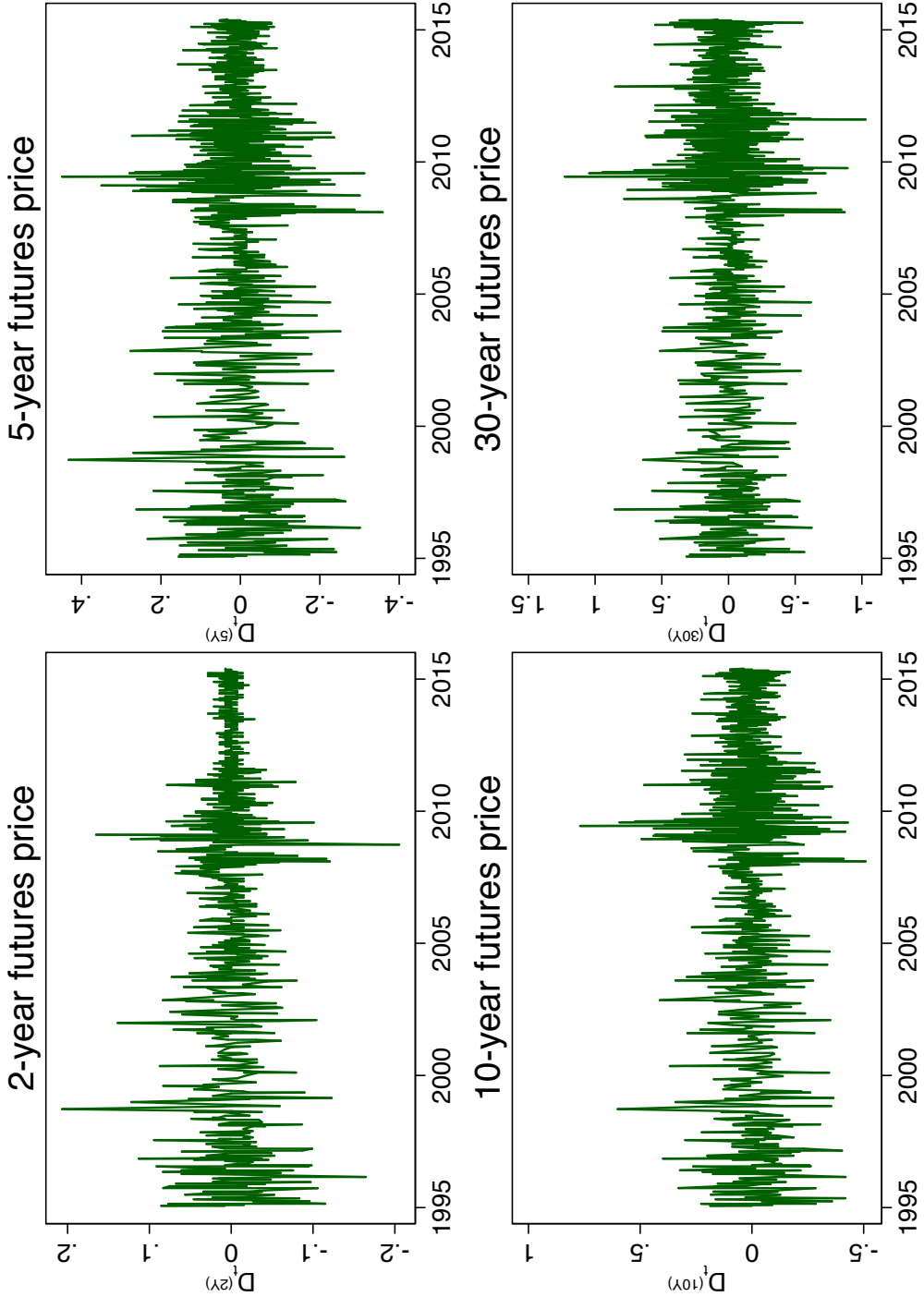
Term and Type of Security	30-Year Bond	
CUSIP Number	912810QS0	
Series	Bonds of August 2041	
Interest Rate	3-3/4%	
High Yield ¹	3.750%	
Allotted at High Price	41.74%	
Accrued Interest per \$1,000	None	
Median Yield ²	3.629%	
Low Yield ³	3.537%	
Issue Date	August 15, 2011	
Maturity Date	August 15, 2041	
Original Issue Date	August 15, 2011	
Dated Date	August 15, 2011	
	Tendered	Accepted
Competitive	\$33,305,800,000	\$15,985,160,000
Noncompetitive	\$14,855,600	\$14,855,600
FIMA (Noncompetitive)	\$0	\$0
Subtotal⁴	<u>\$33,320,655,600</u>	<u>\$16,000,015,600⁵</u>
SOMA	\$489,928,400	\$489,928,400
Total	<u>\$33,810,584,000</u>	<u>\$16,489,944,000</u>
	Tendered	Accepted
Primary Dealer ⁶	\$23,734,000,000	\$10,921,532,000
Direct Bidder ⁷	\$6,567,000,000	\$3,119,654,000
Indirect Bidder ⁸	\$3,004,800,000	\$1,943,974,000
Total Competitive	<u>\$33,305,800,000</u>	<u>\$15,985,160,000</u>

Figure 5: Number of Auctions per Year



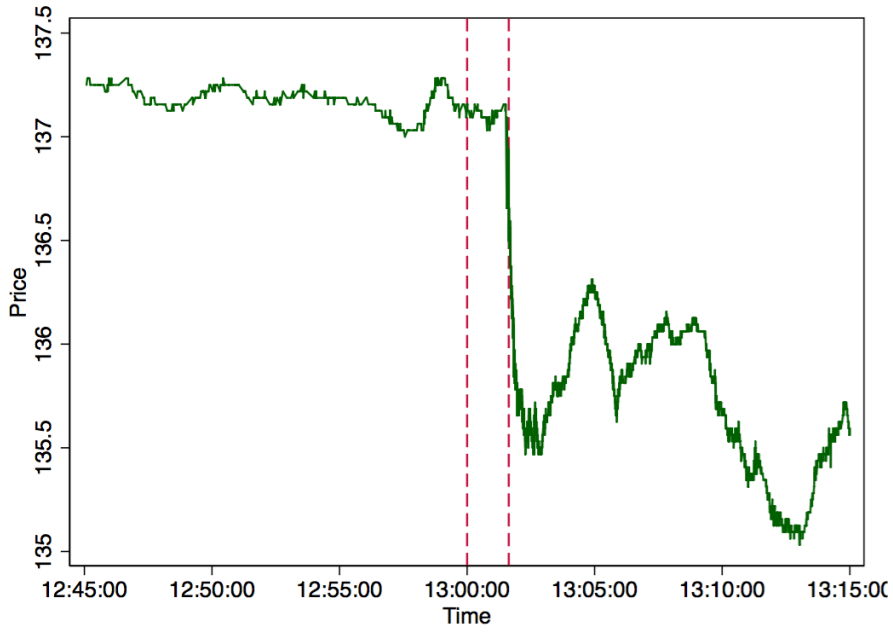
Notes: Number of note and bond Treasury auctions per year by term length in our dataset. The number of auctions temporarily fell in the late 1990s and early 2000s, before increasing sharply after the Great Recession.

Figure 6: Time series of surprises in Treasury futures

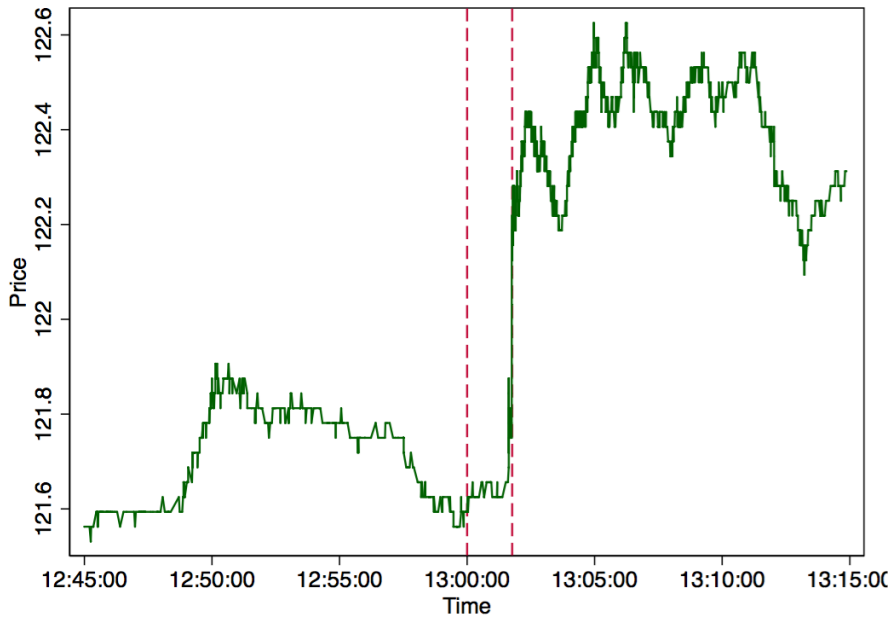


Notes: The figure plots time series of surprise movements in Treasury futures with maturities 2, 5, 10 and 30 years. The movements are reported in log points.

Figure 7: 30-year Auctions

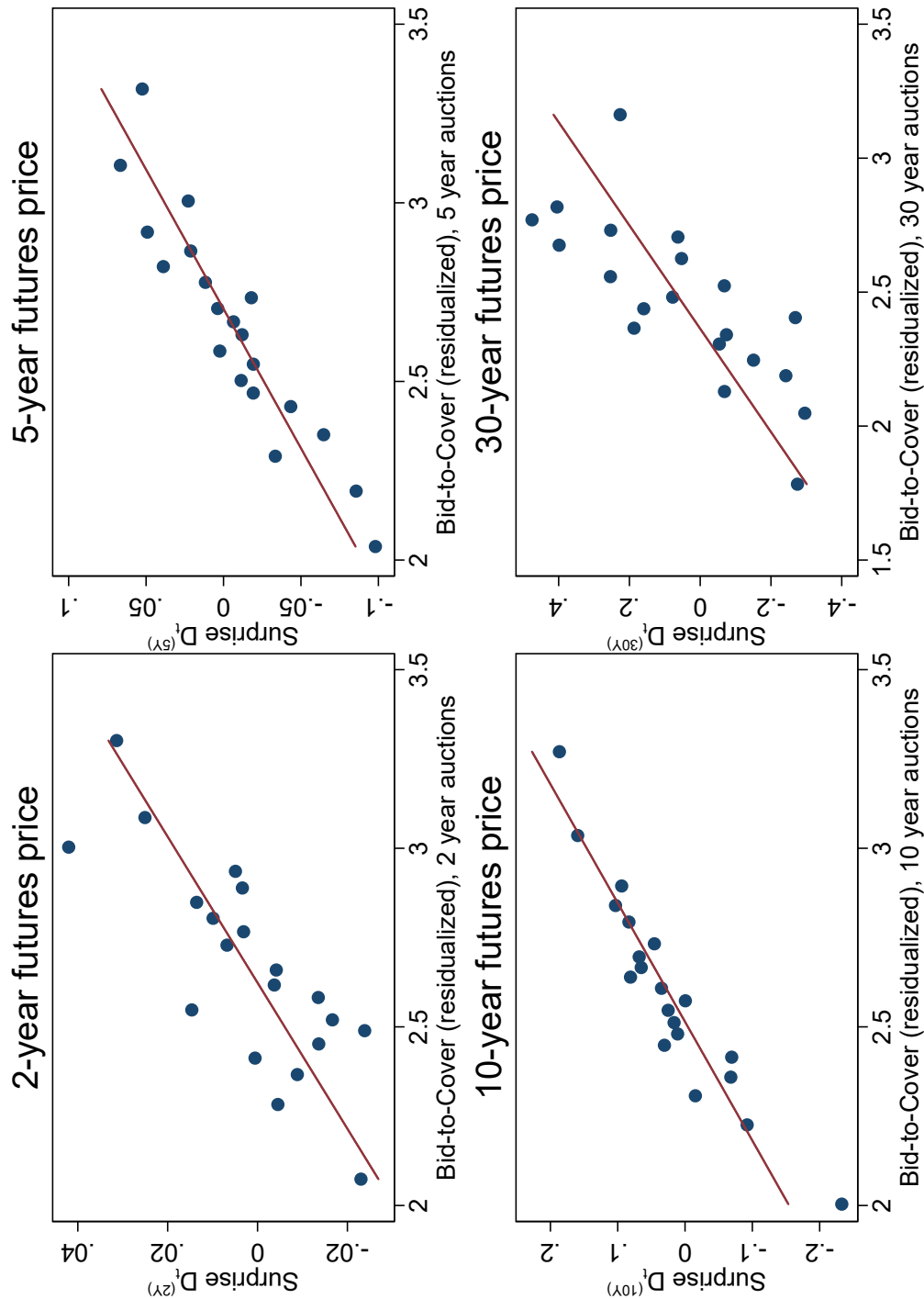


Notes: 30-year Treasury futures prices on August 11, 2011. An auction for 30-year Treasury bonds closed at 1:00pm (first vertical line), and results were released shortly after (second vertical line). Immediately following the release, Treasury futures prices dropped sharply.



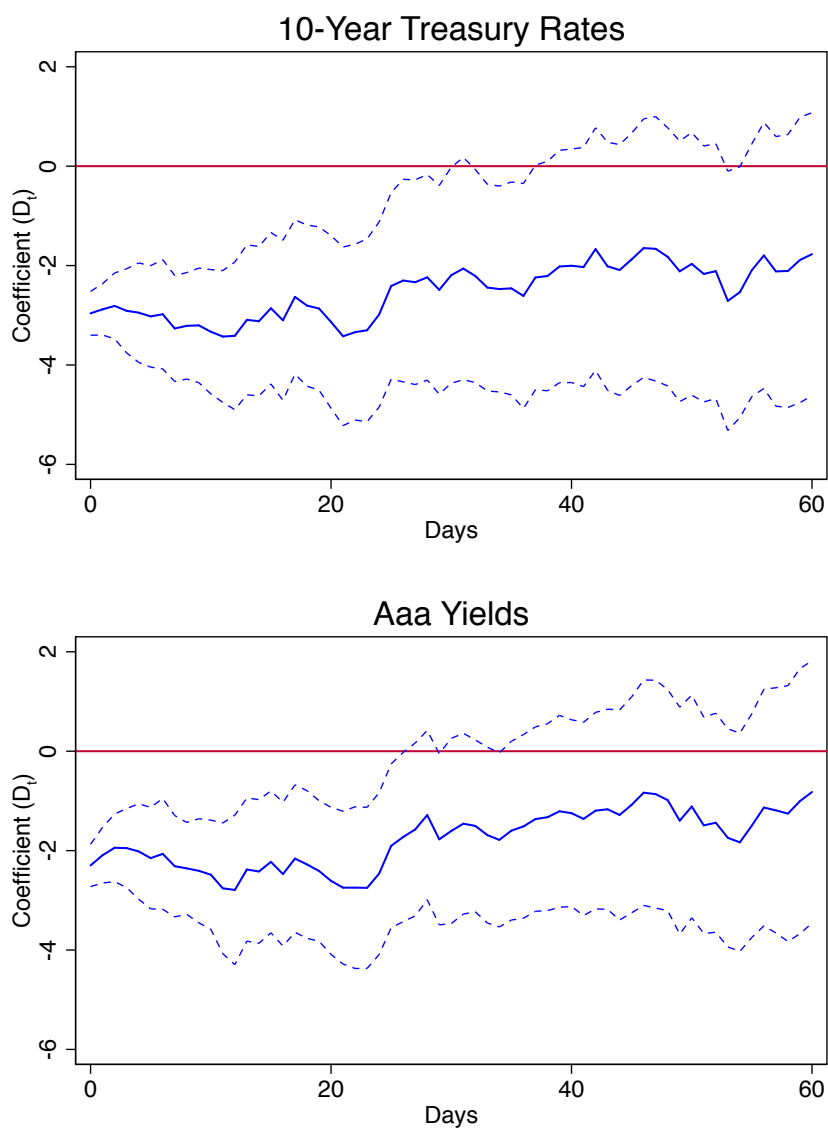
Notes: 30-year Treasury futures prices on December 9, 2010. An auction for 30-year Treasury bonds closed at 1:00pm (first vertical line), and results were released shortly after (second vertical line). Immediately following the release, Treasury futures prices rose sharply.

Figure 8: Demand Shocks and Bid-to-Cover



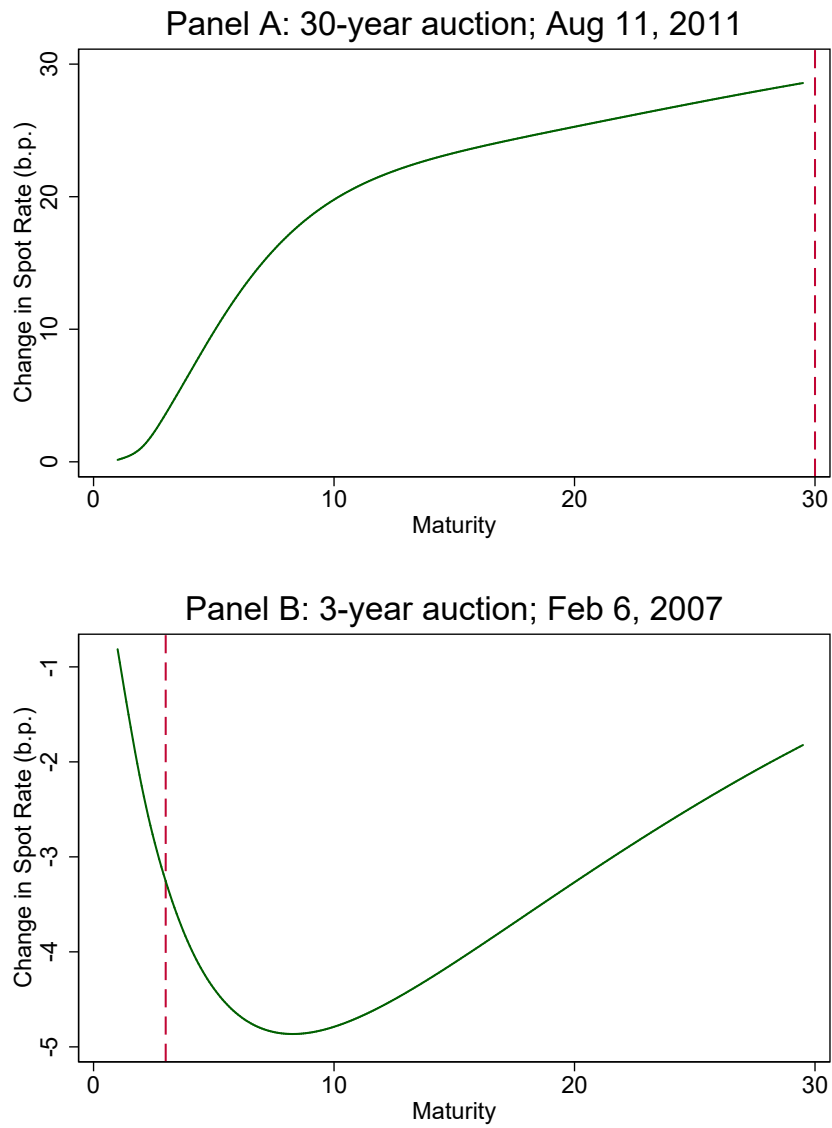
Notes: Bin scatter plot comparing demand shocks D_t and the bid-to-cover ratio from the auction (winsorized at 1% level). The bid-to-cover ratio is the fraction of dollar value of bids received at a given auction. Demand shocks D_t are reported in log points.

Figure 9: Long-Difference Responses to Shock D_t



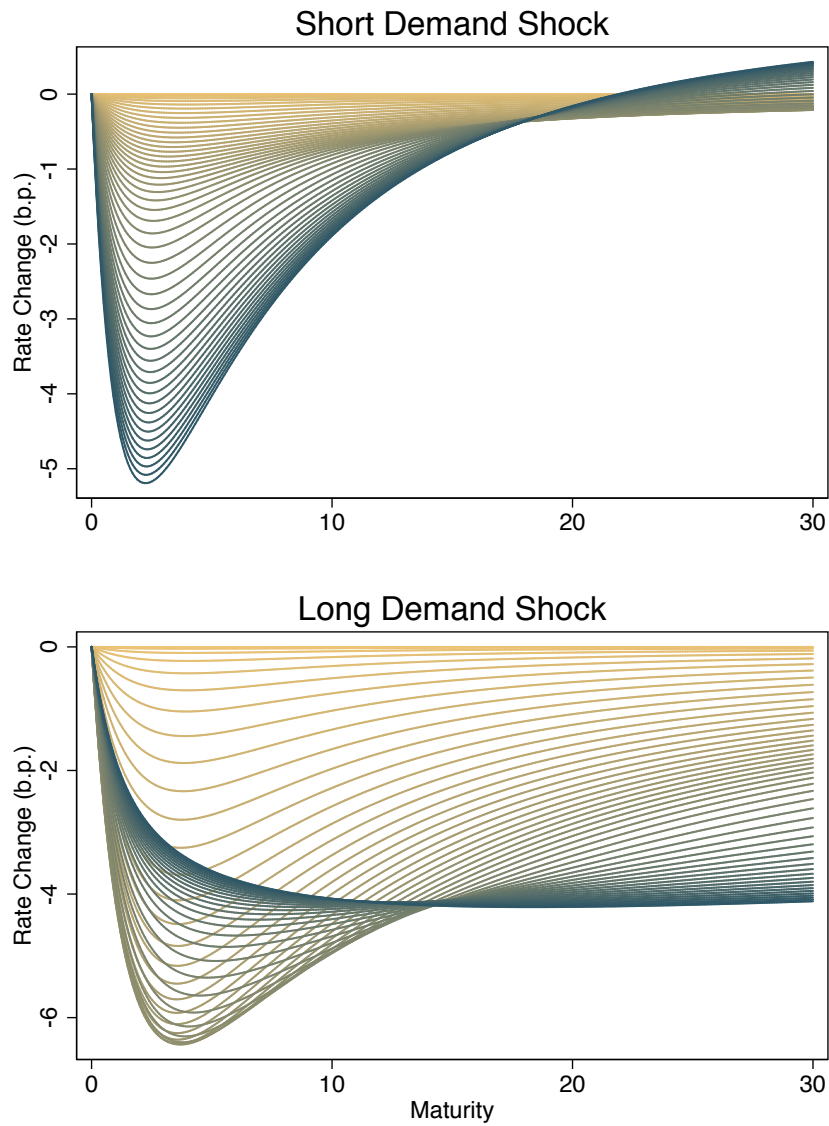
Notes: responses of 10-year Treasury spot rates (top panel) and Moody's Aaa yields (bottom panel) to a unit shock in the first principal component D_t . Spot rates come from [Gürkaynak et al. \(2007\)](#), estimated from daily prices from the secondary market for Treasuries. The regressions are "long-difference" regressions: on an auction date t , the dependent variable is $R_{t+h} - R_{t-1}$, i.e. the change (in terms of basis points) h days after the auction relative to the day before the auction. We plot the coefficients from regressions for $h = 0, \dots, 60$. The solid line plots the point estimates, while dashed lines plot two-standard deviation (Newey-West) confidence bands.

Figure 10: Changes in yield curves on select Treasury auction days



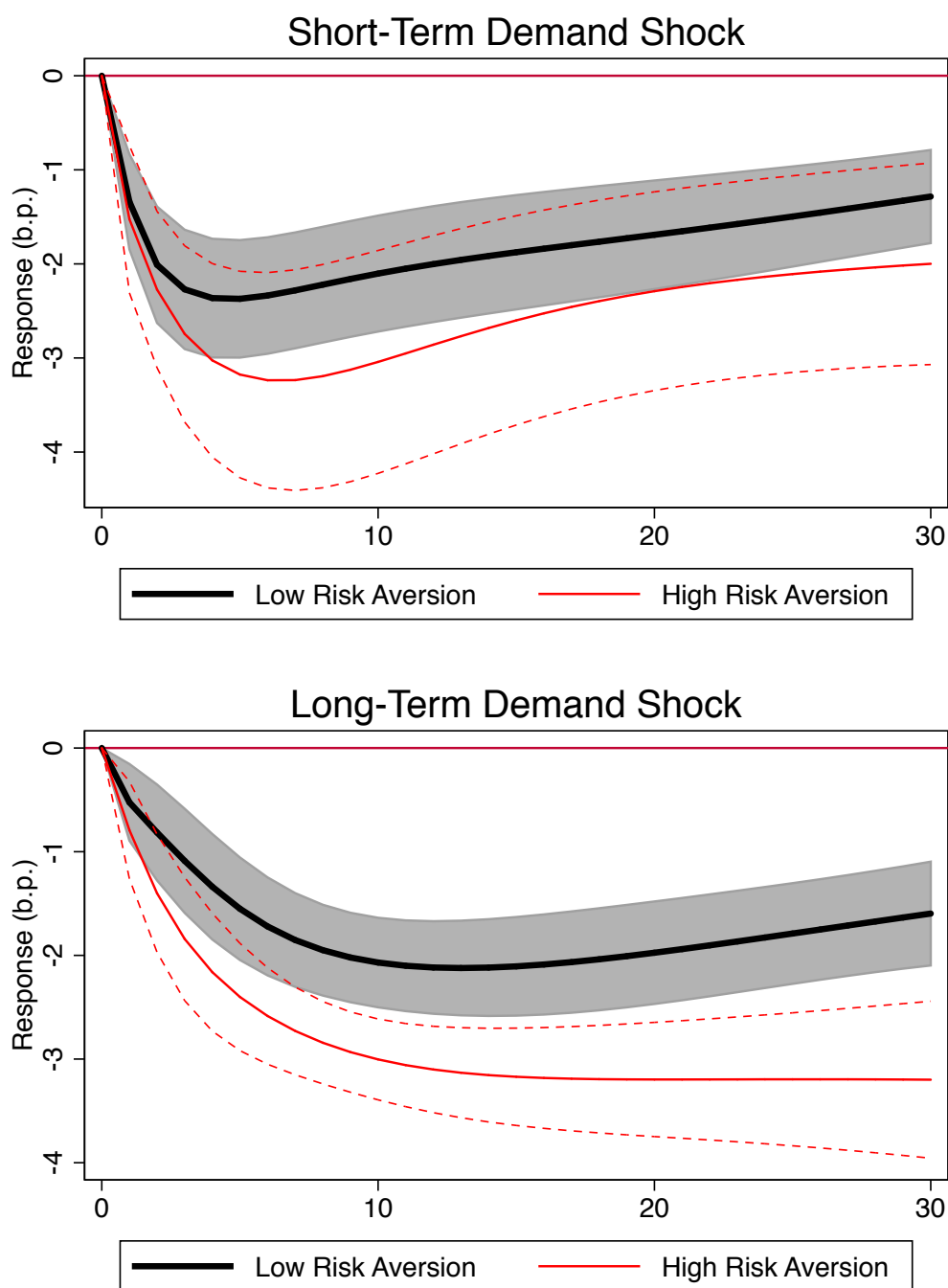
Notes: the figure plots changes in spot rates after 30-year auction on August 11, 2011 (top panel) and 3-year auction on February 6, 2007. The dashed vertical line shows the “location” of the auction in the maturity space.

Figure 11: Numerical Exercise



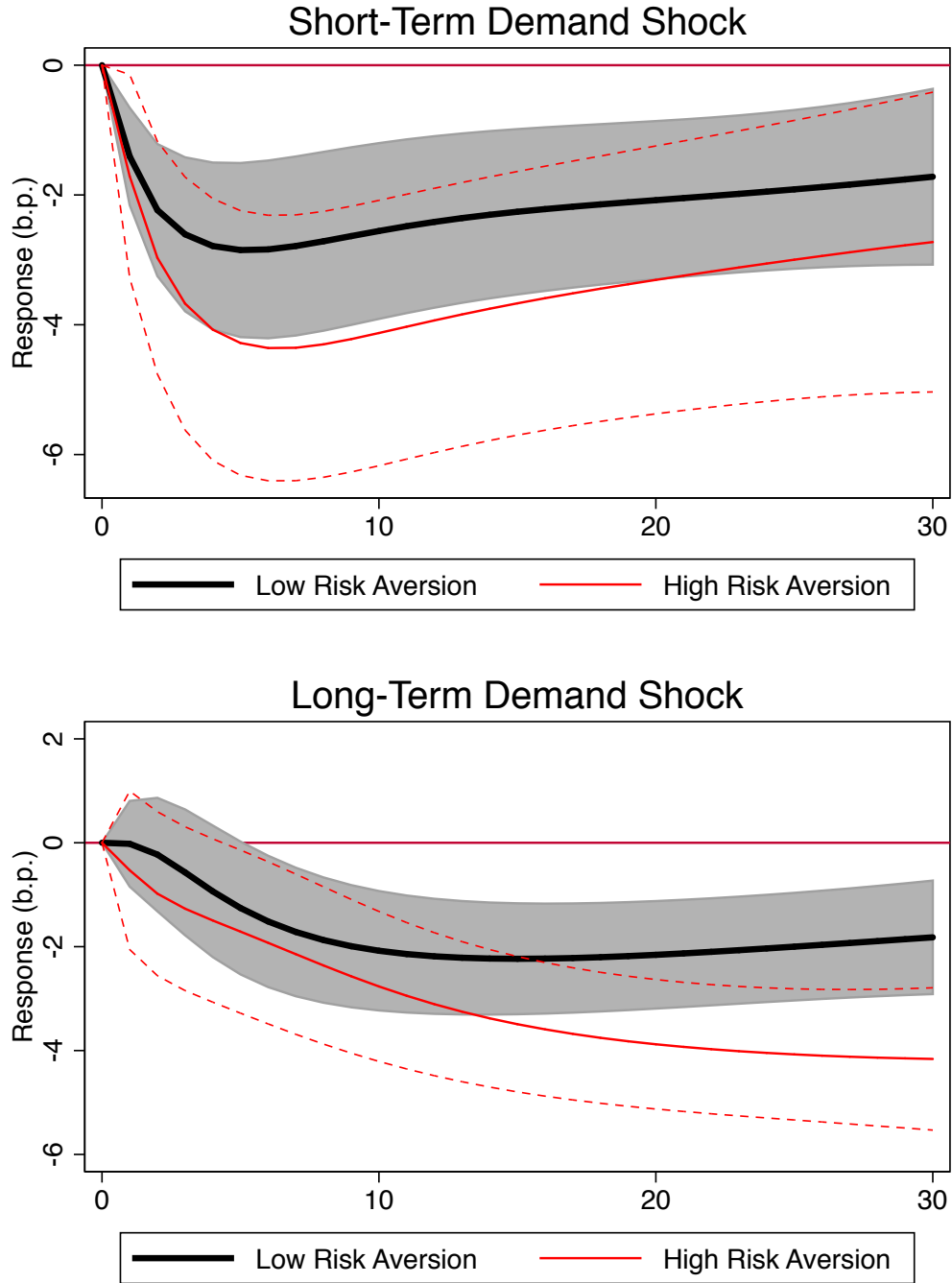
Notes: Numerical exercise studying the change in term structure of spot rates in response to one-standard deviation positive demand shocks, as risk aversion increase from low (lighter) to high (darker). The top panel is the impact of a short demand shock, and the bottom panel is the impact of a long demand shock.

Figure 12: Rate Responses (intraday Futures surprises)



Notes: Plots of the regression coefficients on the demand shocks $D_t^{(m')}$ from regression equation (4). For each auction the demand shock $D_t^{(m')}$ is the normalized futures surprise that most closely corresponds to the maturity of the auction (e.g. a 5-year auction corresponds to $D_t^{(5Y)}$). Each curve is from the subsample combinations: short-term and long-term auctions; and periods of high and low risk aversion. 2 standard error (Newey-West) confidence intervals are included.

Figure 13: Rate Responses (IV specification)



Notes: Plots of the regression coefficients on the demand shocks $D_t^{(m')}$ from regression equation (4), instrumented by the surprise component of the bid-to-cover ratio. Each curve is from the subsample combinations: short-term and long-term auctions; and periods of high and low risk aversion. The first-stage F-statistic for the short-term auctions is 10.16, while for the long-term auctions the F-statistic is 17.58. 2 standard error (Newey-West) confidence intervals are included.

Table 1: Auction Summary Statistics

Panel A: 1979-2015

	Mean	Median	Std. Dev.	Min	Max
Offering Amount (billions)	17.08	14.00	10.13	1.50	44.00
Total Tendered (billions)	47.44	36.38	31.97	2.37	160.96
Bid-to-Cover	2.60	2.57	0.52	1.22	5.88
Term (Years)	7.46	5.00	8.08	2.00	30.25
High Yield	5.39	4.77	3.66	0.22	16.28
High-Median Spread	0.03	0.03	0.02	0.00	0.14

Panel B: 1995-2015

	Mean	Median	Std. Dev.	Min	Max
Offering Amount (billions)	22.03	21.00	9.36	5.00	44.00
Total Tendered (billions)	61.46	52.98	32.04	11.35	160.96
Term (Years)	7.83	5.00	8.42	2.00	30.25
High Yield	3.26	3.20	1.91	0.22	7.79
High-Median Spread	0.03	0.03	0.02	0.00	0.13
Bid-to-Cover	2.62	2.60	0.49	1.22	4.07
Bid-to-Cover by type [#]					
Direct Bidders	0.24	0.25	0.18	0.00	0.84
Indirect Bidders	0.50	0.50	0.16	0.03	1.02
Primary Dealers	1.98	1.92	0.35	0.97	3.12
Fraction Accepted [†]					
Depository Institutions	0.01	0.00	0.02	0.00	0.32
Individuals	0.01	0.00	0.02	0.00	0.19
Dealers	0.58	0.58	0.14	0.20	0.98
Pensions	0.00	0.00	0.01	0.00	0.21
Investment Funds	0.20	0.18	0.13	0.00	0.64
Foreign	0.20	0.19	0.09	0.00	0.61
Other	0.00	0.00	0.00	0.00	0.03

Notes: Summary statistics for Treasury note and bond auctions. † indicates that the moments are computed for 2000-2015, the period for which these data are available. # indicates that the moments are computed for 2003-2015, the period for which these data are available.

Table 2: Treasury Futures Shocks Summary Statistics

Maturity	Mean	Med.	Std. Dev.	N	Correlations			
					$D_t^{(2Y)}$	$D_t^{(5Y)}$	$D_t^{(10Y)}$	$D_t^{(30Y)}$
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Panel A. Auction								
$D_t^{(2Y)}$	-0.000	0.000	0.034	871	1.000			
$D_t^{(5Y)}$	0.002	0.000	0.092	871	0.866	1.000		
$D_t^{(10Y)}$	0.007	0.007	0.143	871	0.782	0.958	1.000	
$D_t^{(30Y)}$	0.006	0.000	0.245	871	0.672	0.848	0.922	1.000
Panel B. No auction								
$D_t^{(2Y)}$	-0.000	0.000	0.031	4031	1.000			
$D_t^{(5Y)}$	-0.001	0.000	0.072	4096	0.862	1.000		
$D_t^{(10Y)}$	-0.002	0.000	0.107	4100	0.794	0.945	1.000	
$D_t^{(30Y)}$	-0.005	0.000	0.172	4099	0.674	0.830	0.905	1.000
Panel C. Auction, non-ZLB period								
$D_t^{(2Y)}$	-0.002	0.000	0.043	424	1.000			
$D_t^{(5Y)}$	-0.005	-0.007	0.099	424	0.922	1.000		
$D_t^{(10Y)}$	-0.005	0.000	0.143	424	0.866	0.962	1.000	
$D_t^{(30Y)}$	-0.016	-0.026	0.223	424	0.778	0.878	0.933	1.000
Panel D. Auction, ZLB period								
$D_t^{(2Y)}$	0.002	0.000	0.022	447	1.000			
$D_t^{(5Y)}$	0.009	0.007	0.083	447	0.811	1.000		
$D_t^{(10Y)}$	0.018	0.014	0.143	447	0.736	0.960	1.000	
$D_t^{(30Y)}$	0.027	0.023	0.263	447	0.642	0.840	0.918	1.000

Notes: On auction dates, shocks $D_t^{(m)} = \log P_{t,post}^{(m)} - \log P_{t,pre}^{(m)}$ are the log intraday change in Treasury futures prices before and after the close of an auction, for each contract $m = 2, 5, 10, 30$ years. For non-auction dates, the shocks are the log intraday changes in Treasury futures prices using the same window. Binding zero lower bound (ZLB) period covers 2008M12-2015M12. Non-ZLB period covers 1995M1-2008M11. Statistics in columns (1)-(3) are reported in log points.

Table 3: Demand Shocks and Measures of Demand

Panel A: Total bid-to-cover ratio

	(1) $D_t^{(2Y)}$	(2) $D_t^{(5Y)}$	(3) $D_t^{(10Y)}$	(4) $D_t^{(30Y)}$	(5) Pool D_t
Bid-to-Cover	1.441*** (0.240)	1.399*** (0.230)	2.099*** (0.216)	2.119*** (0.565)	1.645*** (0.142)
Observations	238	306	227	100	871
R^2	0.156	0.201	0.302	0.270	0.215

Panel B: Expected and Unexpected bid-to-cover ratio

	(1) $D_t^{(2Y)}$	(2) $D_t^{(5Y)}$	(3) $D_t^{(10Y)}$	(4) $D_t^{(30Y)}$	(5) Pool D_t
Bid-to-Cover (exp.)	0.031 (0.113)	-0.041 (0.120)	-0.454* (0.239)	-1.374 (1.654)	-0.076 (0.081)
Bid-to-Cover (unexp.)	1.382*** (0.242)	1.374*** (0.236)	2.113*** (0.216)	2.157*** (0.634)	1.645*** (0.142)
Observations	238	306	227	100	871
R^2	0.124	0.189	0.294	0.215	0.198

Panel C: Total bid-to-cover ratio by bidder type

Indirect Bidder	2.716*** (0.366)	3.664*** (0.667)	4.528*** (0.493)	8.532*** (1.235)	4.451*** (0.436)
Direct Bidder	2.236** (1.034)	1.026 (0.702)	0.295 (0.956)	1.145 (0.951)	1.173*** (0.448)
Primary Dealer	0.831** (0.387)	0.762** (0.316)	1.517*** (0.317)	0.057 (0.536)	0.887*** (0.178)
Observations	138	228	187	80	633
R^2	0.350	0.309	0.383	0.650	0.370

Panel D: Fraction accepted by bidder type

Investment Funds	4.800*** (0.908)	3.401*** (0.854)	4.563*** (0.902)	6.436*** (1.462)	4.749*** (0.494)
Foreign	2.797** (1.162)	3.604*** (0.847)	5.173*** (1.220)	7.974*** (2.404)	4.393*** (0.676)
Misc	4.815* (2.614)	2.506** (1.203)	0.034 (3.713)	0.853 (5.119)	2.353** (1.193)
Observations	174	241	201	84	700
R^2	0.214	0.128	0.287	0.391	0.191

Notes: Regressions of demand shocks $D_t^{(m)}$ on the bid-to-cover ratio, total and broken up by bidder type (winsorized at 1% level). Four lags of bid-to-cover ratios (or fractions accepted) are included but not reported. Column (1) restricts the sample to include only auctions of 2-year notes. Column (2) restricts the sample to include only auctions of notes with (2,5] year maturity. Column (3) restricts the sample to include only auctions of notes with [7,10] year maturity. Column (4) restricts the sample to include only auctions of bonds with (10,30] year maturity. Shocks $D_t^{(m)}$ are standardized to have zero mean and unit variance. Newey-West standard errors in parentheses.

Table 4: Reaction of market to surprises at Treasury auctions

Dep.variable: asset type	Estimate (s.e.)	N	R^2	Sample
	(1)	(2)	(3)	(4)
Panel A. Debt				
TLT	0.312*** (0.016)	662	0.679	2002-2015
SHY	0.022*** (0.001)	662	0.528	2002-2015
LQD	0.110*** (0.008)	662	0.544	2002-2015
Aaa [†]	-2.295*** (0.212)	871	0.173	1995-2015
Panel B. Equities				
SPY	-0.020 (0.018)	871	0.005	1995-2015
IWM	-0.081*** (0.024)	706	0.034	2000-2015
SP500 [†]	-0.072 (0.064)	871	0.004	1995-2015
Russell 2000 [†]	-0.169** (0.069)	871	0.013	1995-2015
Panel C. Inflation expectations and commodities				
10Y Inflation Swap [†]	-0.172 (0.131)	618	0.003	2004-2015
2Y Inflation Swap [†]	0.044 (0.229)	618	0.000	2004-2015
GLD	0.021 (0.015)	595	0.004	2004-2015
GSCI [†]	0.008 (0.056)	871	0.000	1995-2015
Panel D. Spreads and credit default swaps				
Baa-Aaa [†]	-0.056 (0.074)	871	0.001	1995-2015
Auto CDS [†]	-3.254 (5.796)	627	0.000	2004-2015
Bank CDS [†]	0.426 (0.450)	627	0.004	2004-2015
3-month LIBOR-OIS [†]	-0.002 (0.002)	630	0.006	2003-2015
VIX [†]	0.058 (0.082)	871	0.001	1995-2015

Notes: The table reports estimates $\hat{\phi}$ from regression equation (3). Assets with [†] are measured at the daily frequency; other price changes are measured over the intraday window corresponding to what we use to construct surprises. The intraday changes are from ETFs that track various underlying securities or indices: TLT (long-term Treasuries); SHY (short-term Treasuries); LQD (corporate debt); SPY (S&P 500); IWM (Russell 2000); GLD (gold bullion). For the daily series: Aaa and Baa (Moody's corporate debt yields); SP500 (daily equity index); Russell 2000 (daily equity index); GSCI (S&P Total Commodity Index); Auto and Bank CDS (industry credit default swaps indices); LIBOR-OIS (3-month USD LIBOR-Overnight Index Swap spread); VIX (daily implied volatility index). Newey-West standard errors in parentheses.

Appendix A Numerical Exercise Details

In this section we briefly describe the model and calibration of our numerical exercise. For more details regarding the model setup, see [Vayanos and Vila \(2009\)](#).

A.1 Numerical Exercise Model

There is a continuum of zero-coupon bonds with maturities $m \in (0, T]$ in zero net supply. A bond with maturity m has a time t price of $P_{t,m}$ and pays \$1 at time $t + m$. The spot rate is $R_{t,m}$ which is given by

$$R_{t,m} = -\frac{\log P_{t,m}}{m}$$

There are two types of investors: idiosyncratic/clientèle investors and arbitrageurs. By assumption idiosyncratic demand takes the following form:

$$y_{t,m} = \alpha(m)m(R_{t,m} - \beta_{t,m})$$

where $\beta_{t,m}$ is a demand shifter which responds to K demand factors:

$$\beta_{t,m} = \bar{\beta} + \sum_{k=1}^K \theta_k(m)\beta_{k,m}$$

Arbitrageurs choose how much of each bond to hold (denoted by $x_{t,m}$). Their budget constraint is:

$$dW_t = \left(W_t - \int_0^T x_{t,m} dm \right) r_t dt + \int_0^T x_{t,m} \frac{dP_{t,m}}{P_{t,m}} dm$$

where r_t is the instantaneous rate: $\lim_{m \rightarrow 0} R_{t,m} = r_t$. Arbitrageurs maximize an instantaneous mean-variance trade-off:

$$\max_x E_t dW_t - \frac{a}{2} \text{Var}_t dW_t$$

where the parameter a governs the level of risk aversion. In equilibrium, we have $y_{t,m} = -x_{t,m}$.

We assume the instantaneous rate and demand factors are stacked in a $K + 1$ vector \mathbf{Y} which follows an Ornstein-Uhlenbeck process:

$$d\mathbf{Y}_t = -\Gamma(\mathbf{Y}_t - \bar{\mathbf{Y}})dt + \mathbf{S}d\mathbf{B}_t$$

where \mathbf{B}_t is a vector of Brownian motions.

It turns out that the above is consistent with bond prices that are affine in the \mathbf{Y} factors:

$$-\log P_{t,m} = \mathbf{Y}_t^T \mathbf{A}(m) + C(m)$$

We are interested in the response of the term structure to shocks to the demand factors, and hence need to solve the model for the coefficient functions $\mathbf{A}(m)$. Using the arbitrageur FOCs and taking into account the zero net supply condition, these functions must satisfy the system of differential equations

$$\mathbf{A}'(m) + \Gamma^T \mathbf{A}(m) - \mathbf{e}_1 = a\mathbf{M}\mathbf{A}(m)$$

where \mathbf{e}_1 is the first coordinate vector (assuming r_t is ordered first in \mathbf{Y}), and

$$\mathbf{M} = \left(\int_0^T \alpha(m) [m\Theta(m) - \mathbf{A}(m)] \mathbf{A}(m)^T dm \right) \mathbf{S}\mathbf{S}^T$$

Solving the above differential equation is made more difficult by the presence of the integral terms in \mathbf{M} . [Vayanos and Vila \(2009\)](#) solves the model for the limiting case when the risk aversion parameter $a \rightarrow 0$ or $a \rightarrow \infty$ (and the particular case when Γ and \mathbf{S} are diagonal), but for intermediate values a solution must be found numerically. For details regarding the solution algorithm, see [Ray \(2017\)](#).

A.2 Numerical Exercise Calibration

For our numerical exercise, we take the number of demand factors to be $K = 2$. We will interpret the first demand factor as a “short” demand factor denoted by $\beta_{t,s}$. The second factor is taken to be a “long” demand factor denoted by $\beta_{t,\ell}$. We assume

$$\mathbf{\Gamma} = \begin{bmatrix} \kappa_r & 0 & 0 \\ 0 & \kappa_s & 0 \\ 0 & 0 & \kappa_\ell \end{bmatrix}$$

and set these mean reversion parameters to imply that shocks to the instantaneous rate have a half-life of approximately one year, while shocks to the demand factors have a half-life of 2.5 years.

For simplicity we also assume uncorrelated shocks, and that the size of the innovations for each factor is equal, i.e. $\mathbf{S} = \sigma\mathbf{I}$. We set $\sigma = .01$.

$\theta_s(m)$ and $\theta_\ell(m)$ govern where the demand shocks are located in maturity space. Although not realistic, we set these as Dirac delta functions, so that the short demand shock is entirely concentrated at idiosyncratic investors whose habitat is at $m = 3$ years; similarly for the long demand shock we choose $m = 20$ years. These maturities roughly correspond to the average maturity of the short-term and long-term auctions in the empirical counterpart. We could have instead assumed these functions have non-zero values for a continuum of bonds, but still concentrated at the long and short end of the maturity space. This complicates the numerical solution algorithm, but leads to largely similar

results.

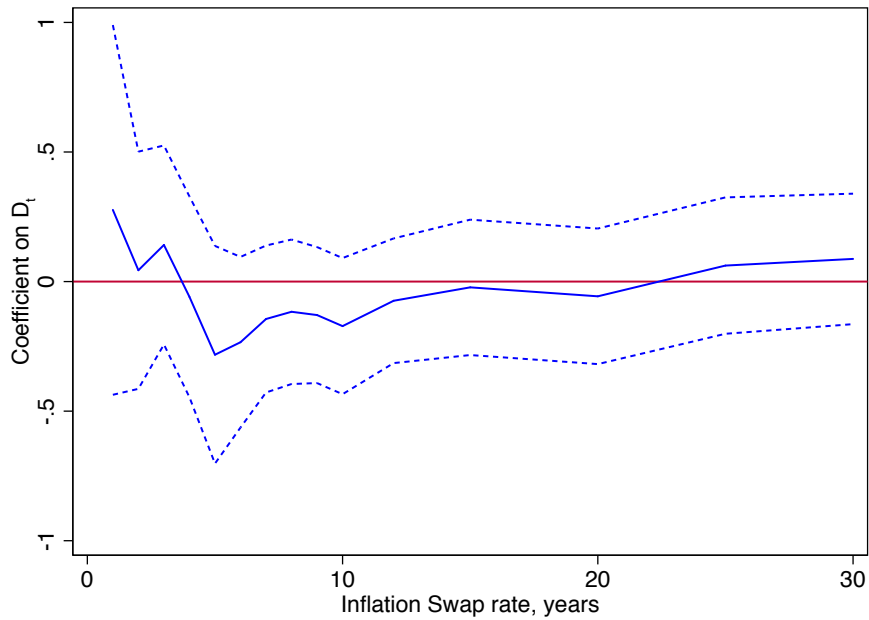
$\alpha(m)$ governs the sensitivity of idiosyncratic investors to changes in the price of bonds within their habitat. We don't have priors for this parameter, and for simplicity assume the function is constant. We set this value to match the following empirical counterpart: a standard deviation increase in our demand shock D_t is associated with an increase of 0.15 in the bid-to-cover ratio during short-term auctions. Given our parameterization above, a one-standard deviation positive short demand shock increases idiosyncratic demand by $3\alpha\sigma$. Equating these values implies $\alpha = 5$.

Finally, we let the risk aversion parameter vary from 0 to 500. The upper limit is ad-hoc; the value was chosen as the response of spot rates at this point begins to stabilize.

Appendix Table B3 summarizes the parameter calibration. The spirit of the numerical exercise is not to match the data perfectly, but rather to gain some qualitative predictions for intermediate levels of risk aversion. Finally, it is important to note that the parameters a , α , and σ in this specification enter multiplicatively. Hence an appropriate rescaling of these values will give numerically identical responses.

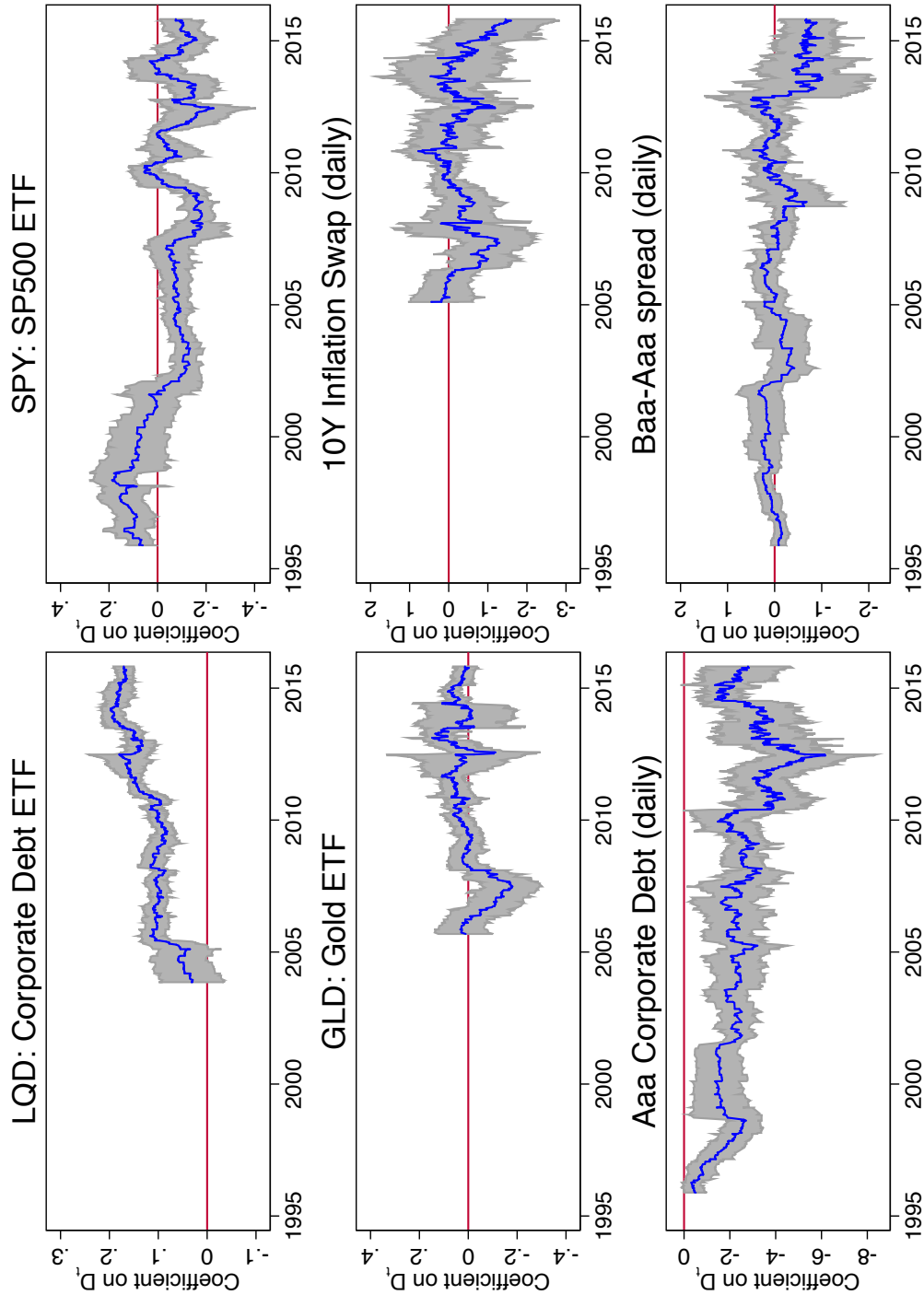
Appendix B Additional Figures and Tables

Figure B1: Response of Inflation Swap Rates to Shock D_t



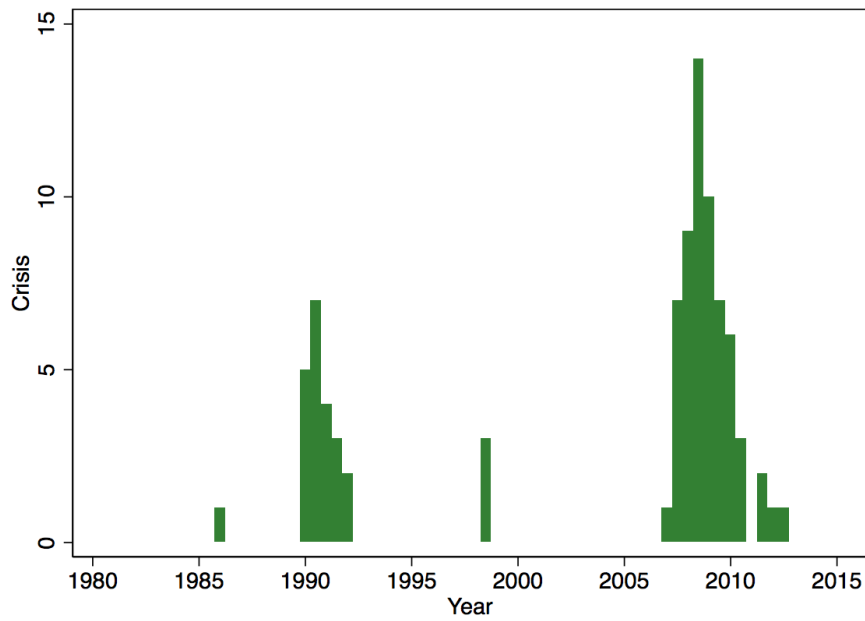
Notes: the figure plots responses of inflation swap rates across different maturities to a shock in the first principal component D_t . The solid line plots the point estimates, while dashed lines plot two-standard deviation (Newey-West) confidence bands.

Figure B2: Rolling Regressions



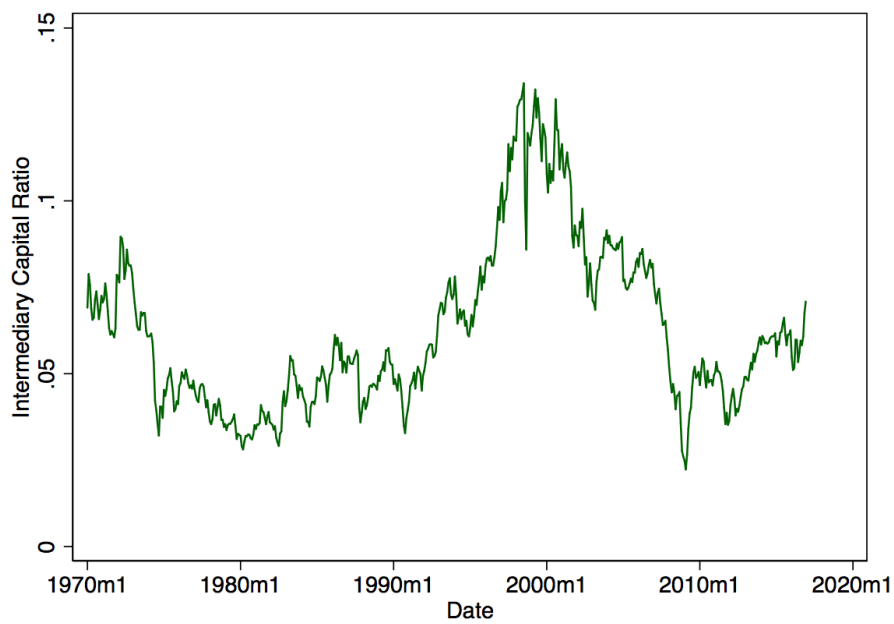
Notes: Coefficient estimates of rolling regressions; each data point is from estimating (3) for using the 60 most recent auctions. LQD, SPY, and GLD are intraday changes; 10-year inflation swap is daily change. Aaa Corporate Debt is the daily change in the interest rate for Aaa corporate bonds. Baa-Aaa Spread is the daily change in the spread between Baa and Aaa corporate bonds. The regressor is always intraday D_t , the first principal component in $D_t^{(2Y)}$, $D_t^{(5Y)}$, $D_t^{(10Y)}$, and $D_t^{(30Y)}$.

Figure B3: U.S. Financial Crises



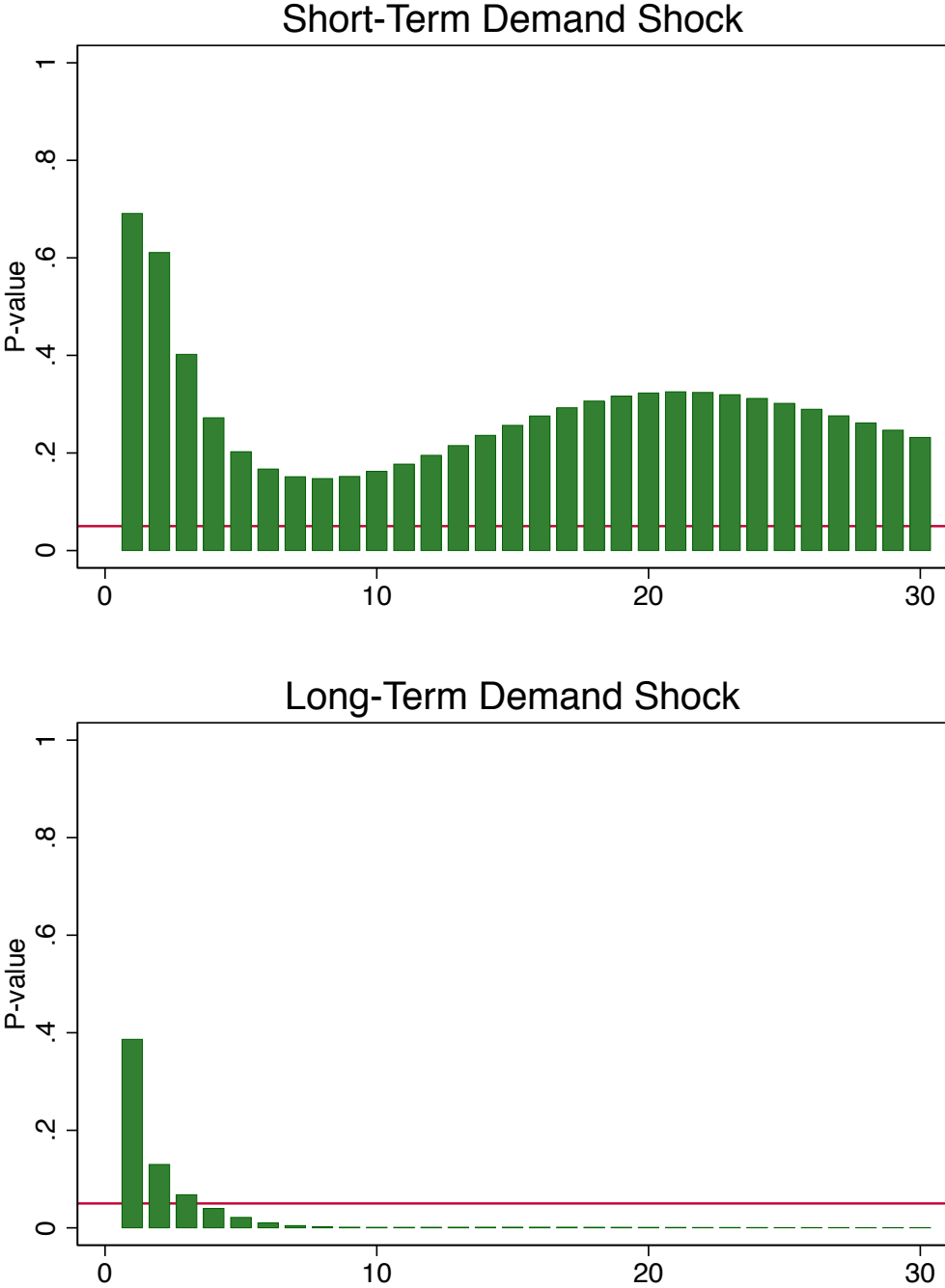
Notes: Financial Crisis indicator for the United States from [Romer and Romer \(2017\)](#).

Figure B4: Intermediary Capital Ratio



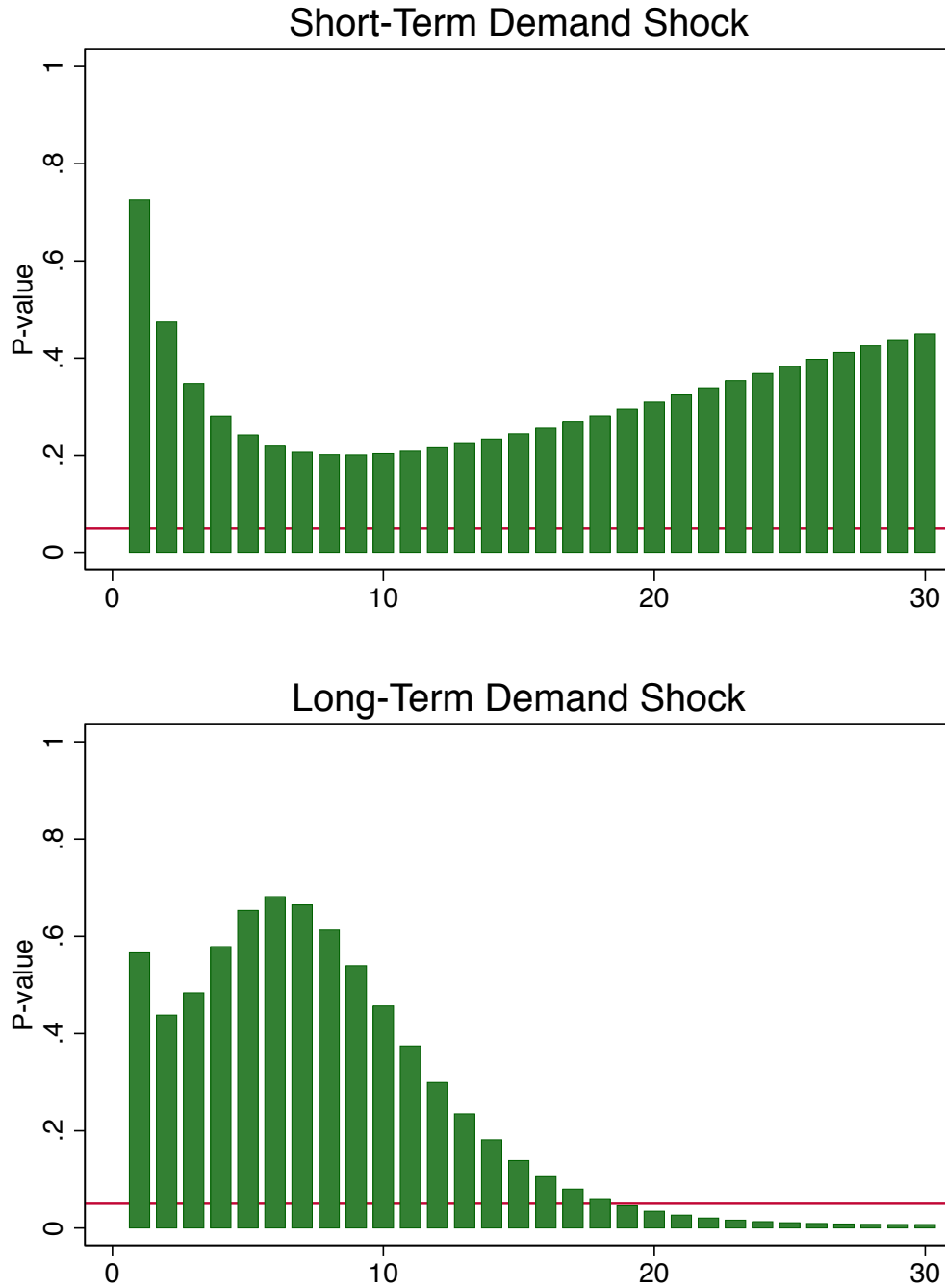
Notes: Intermediary capital ratio from [He et al. \(2016\)](#).

Figure B5: Rate Response P-Values



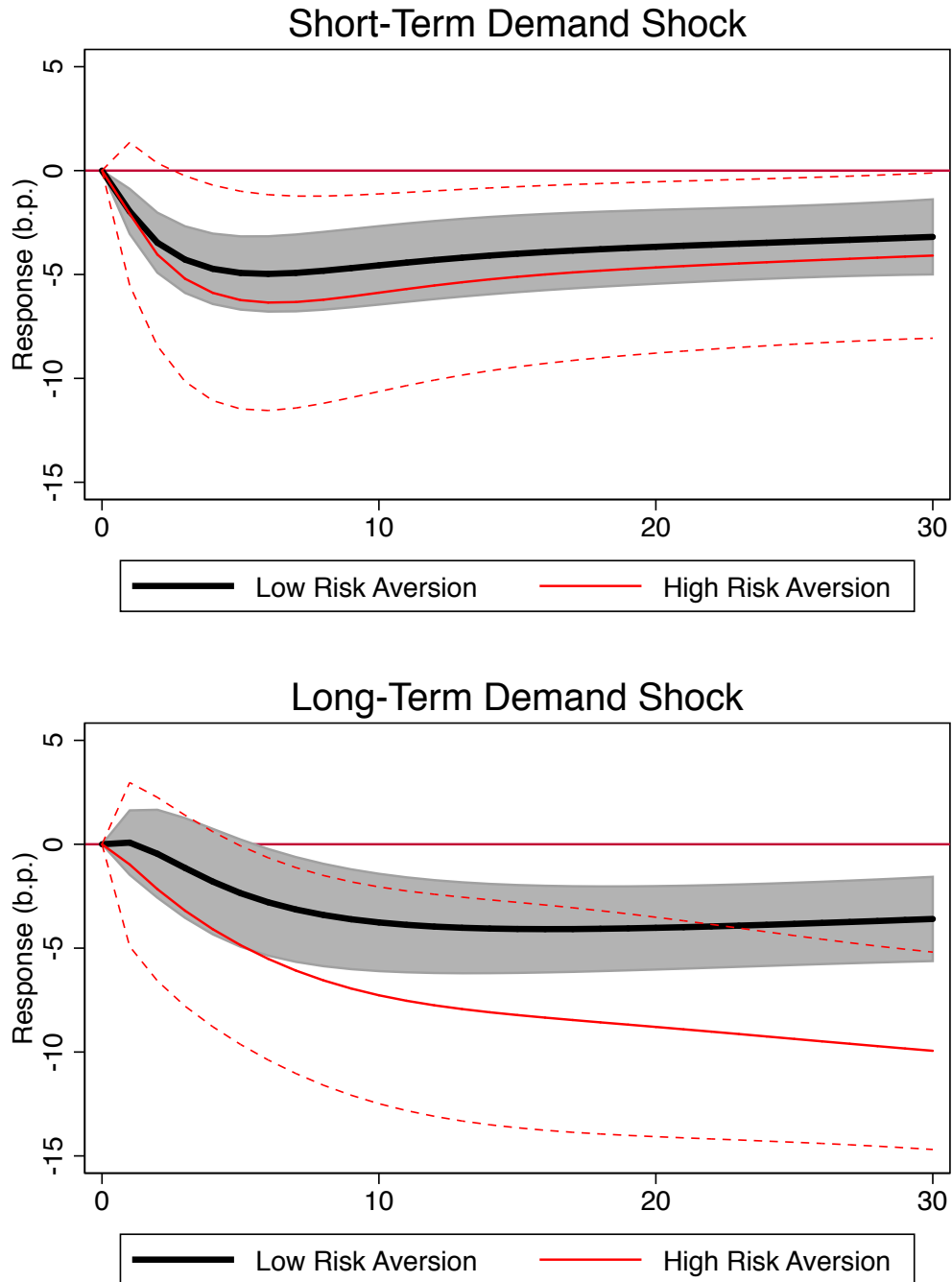
Notes: p-values testing equality of coefficients from Figure 12.

Figure B6: Rate Response P-Values



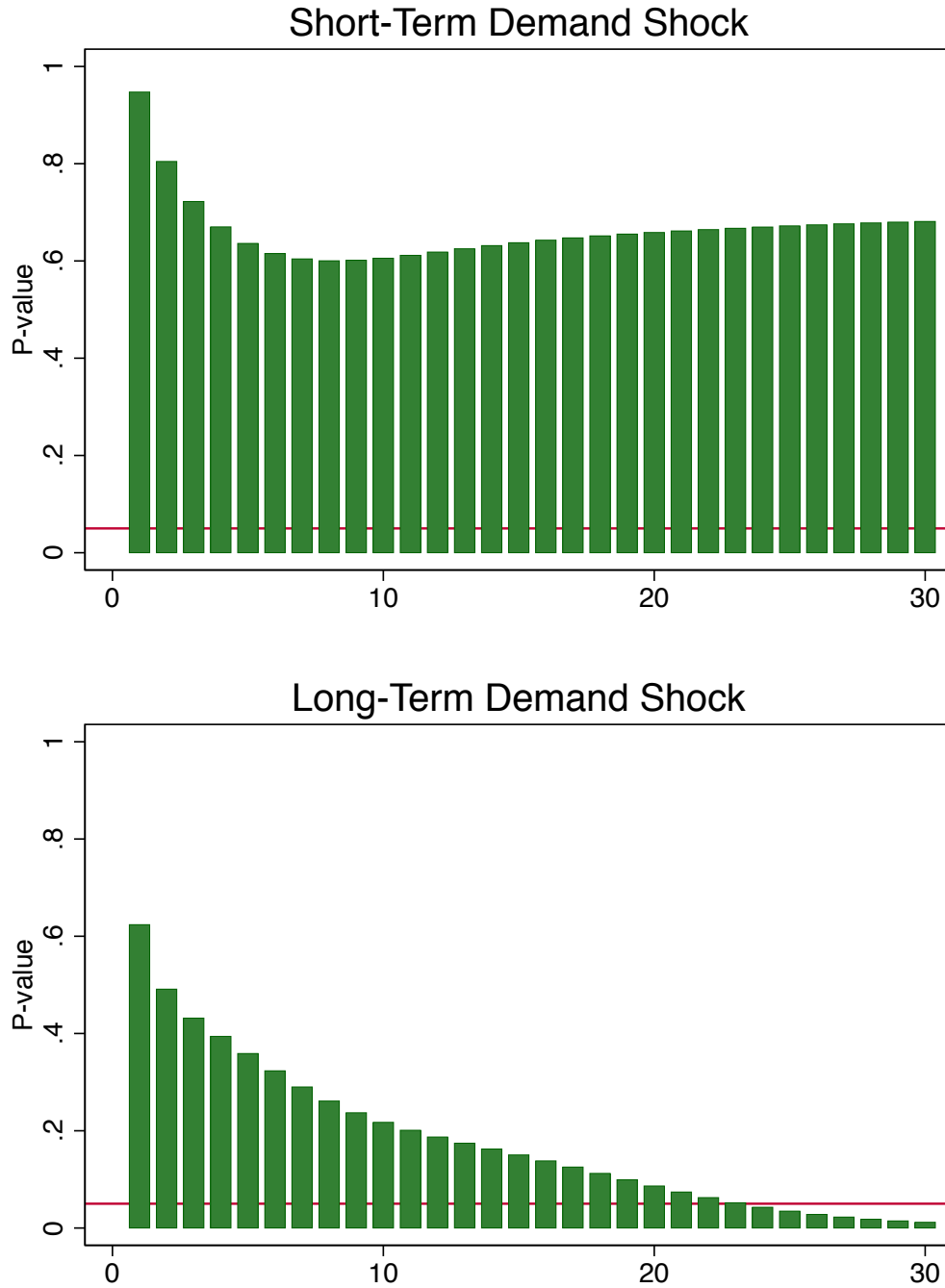
Notes: p-values testing equality of coefficients from Figure 13.

Figure B7: Rate Responses (Bid-to-Cover, 1995-2015)



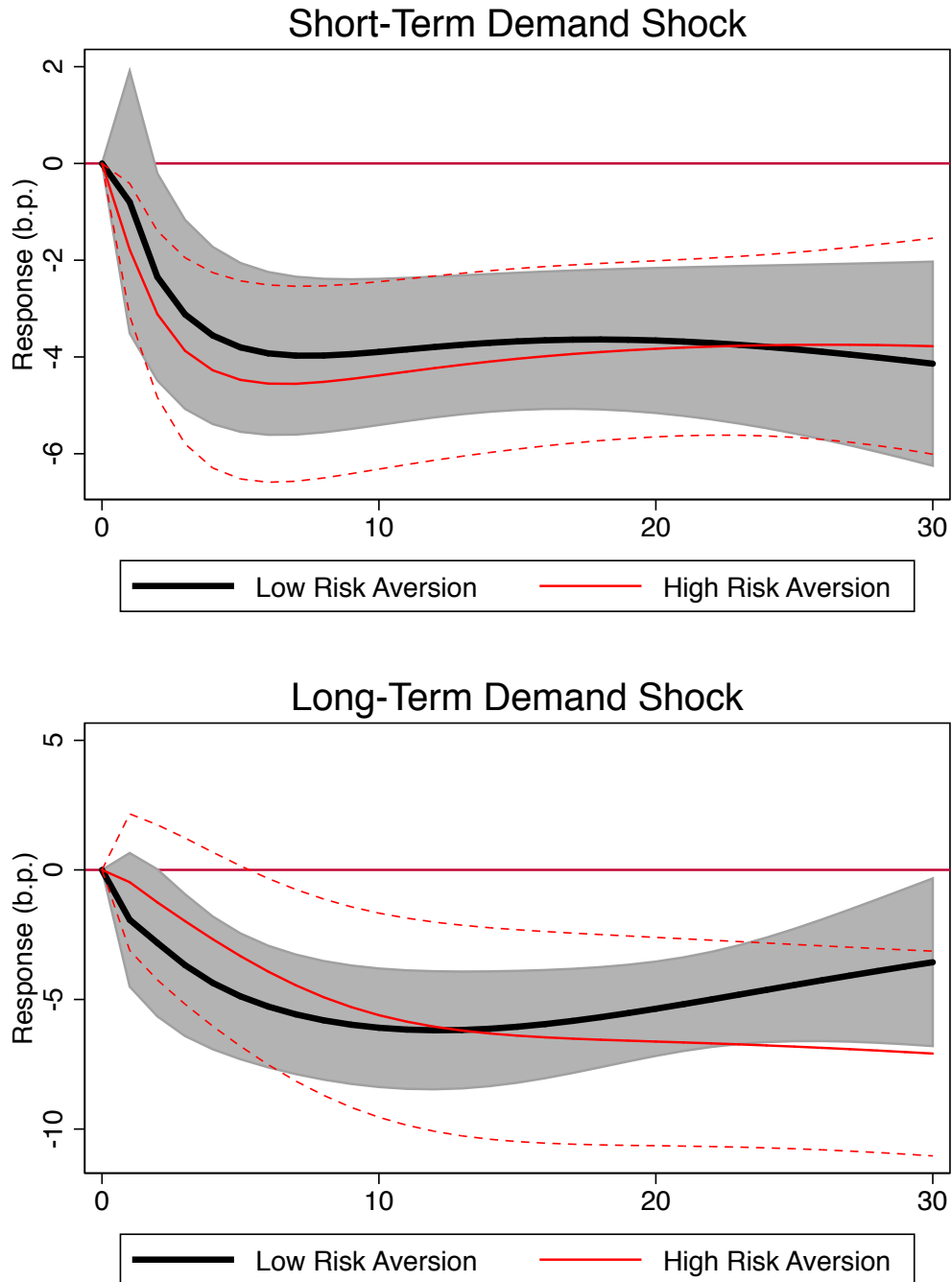
Notes: Plots of the regression coefficient on the surprise component of the bid-to-cover ratio from regression equation (4) for the sample 1995-2015. Each curve is from the subsample combinations: short-term and long-term auctions; and periods of high and low risk aversion as measured. 2 standard error (Newey-West) confidence intervals are included.

Figure B8: Rate Response P-Values (Bid-to-Cover, 1995-2015)



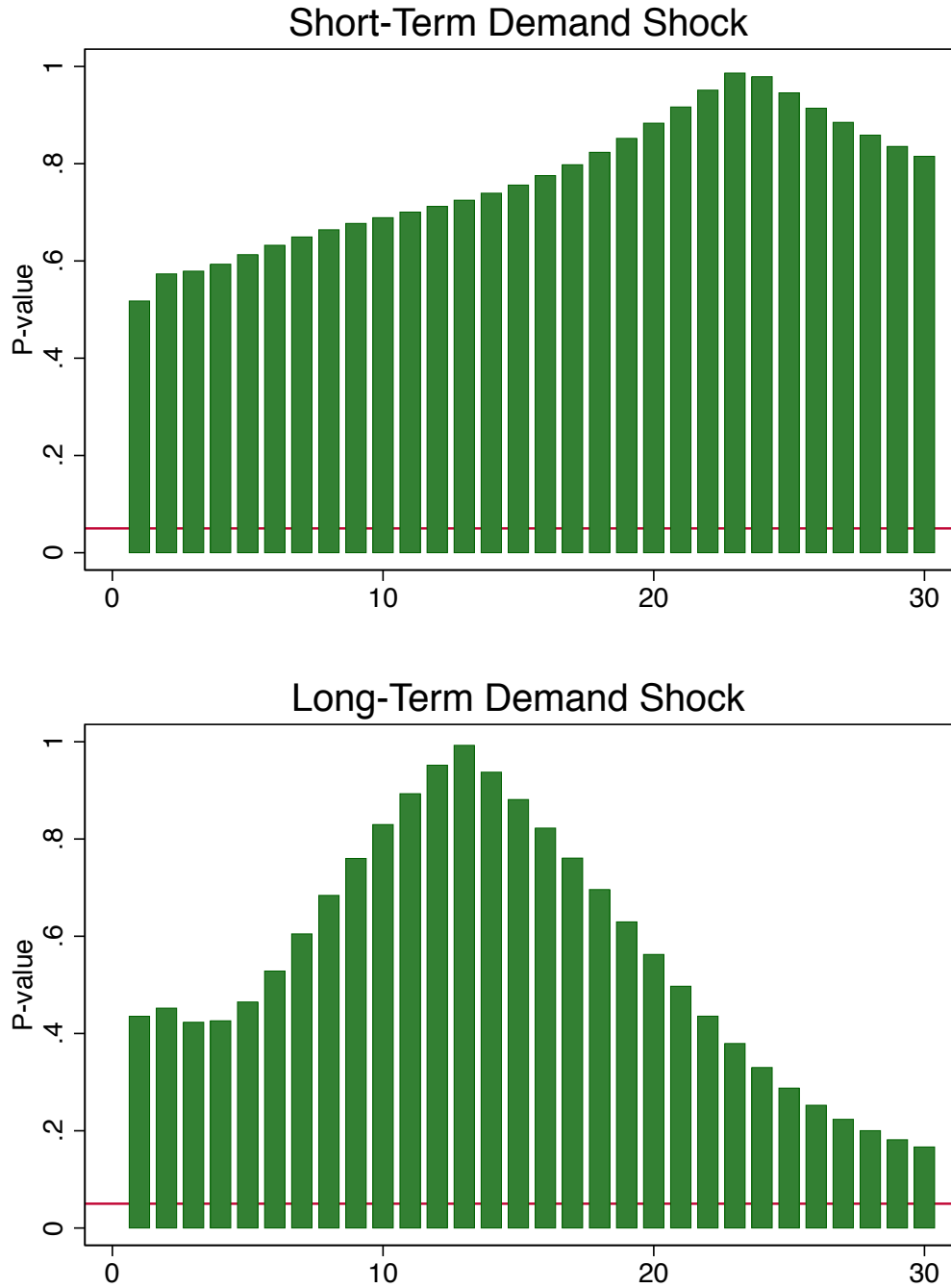
Notes: p-values testing equality of coefficients from Figure B7.

Figure B9: Rate Responses (Bid-to-Cover, 1979-2015)



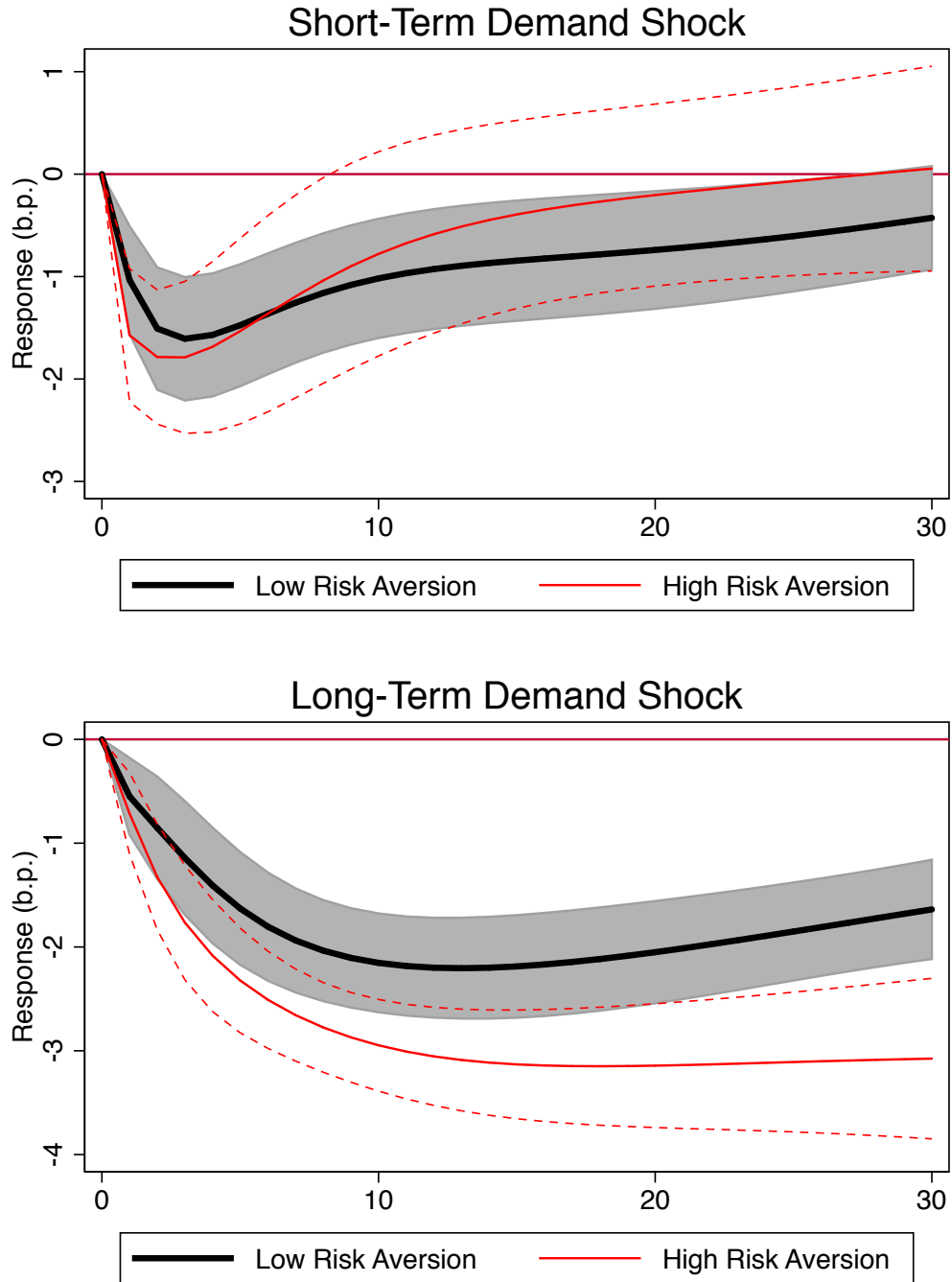
Notes: Plots of the regression coefficient on the surprise component of the bid-to-cover ratio from regression equation (4) for the sample 1979-2015. Each curve is from the subsample combinations: short-term and long-term auctions; and periods of high and low risk aversion as measured. 2 standard error (Newey-West) confidence intervals are included.

Figure B10: Rate Response P-Values (Bid-to-Cover, 1979-2015)



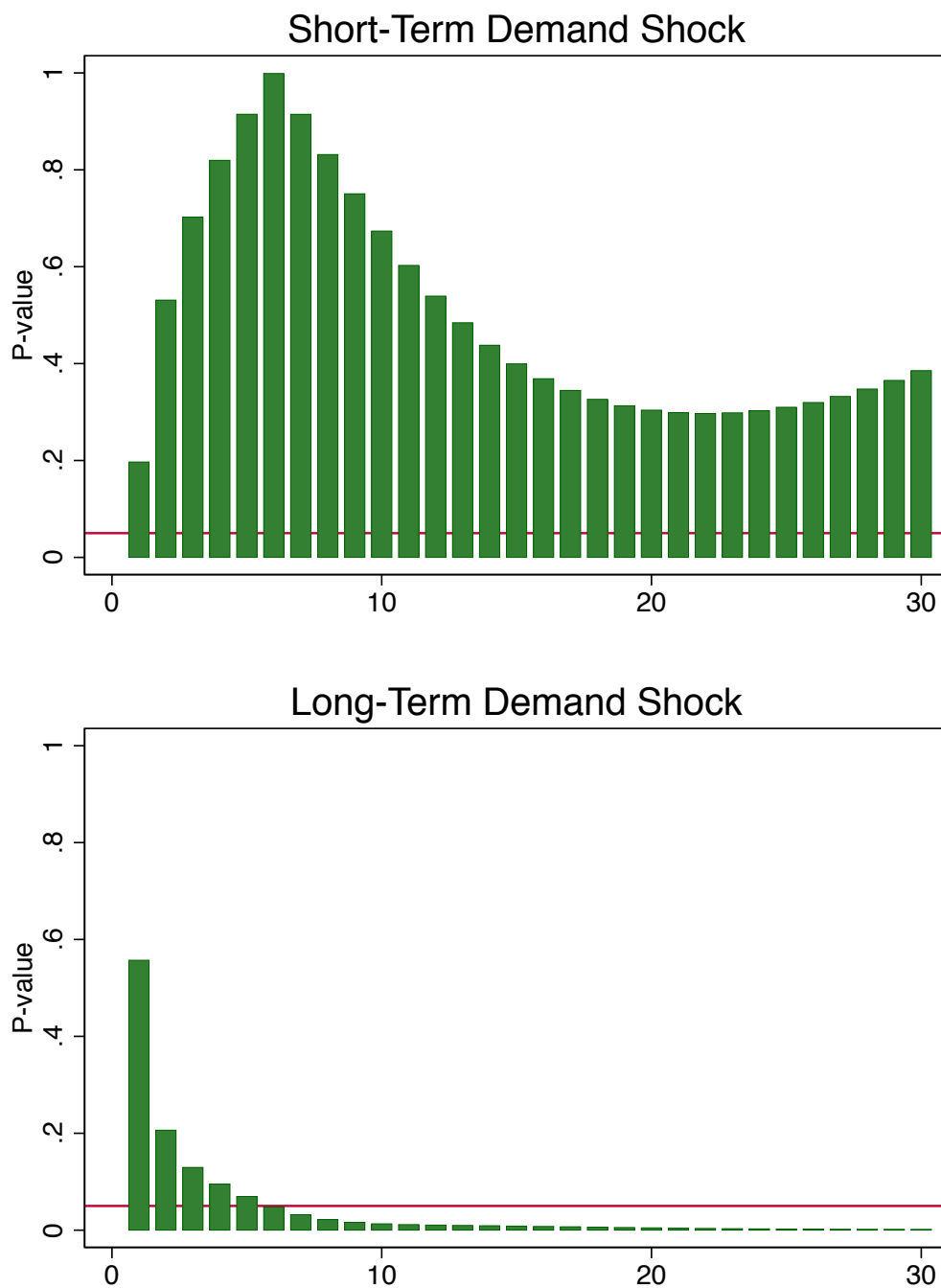
Notes: p-values testing equality of coefficients from Figure B9.

Figure B11: Rate Responses (rotated intraday Futures surprises)



Notes: Plots of the regression coefficients on the demand shocks D_t^i from regression equation (4). The shocks are the first two principal components of our intraday shocks, D_t^ℓ and D_t^s , rotated such that D_t^s is uncorrelated with $D_t^{(30Y)}$. For long-term auctions the shock is D_t^ℓ ; similarly short-term auctions use D_t^s . Each curve is from the subsample combinations: short-term and long-term auctions; and periods of high and low risk aversion. 2 standard error (Newey-West) confidence intervals are included.

Figure B12: Rate Response P-Values (rotated intraday Futures surprises)



Notes: p-values testing equality of coefficients from Figure B11.

Table B1: Reaction of market to surprises at Treasury auctions (IV specification)

Dep.variable: asset type	Estimate (s.e.)	N	F-stat	Sample
	(1)	(2)	(3)	(4)
Panel A. Debt				
TLT	0.359*** (0.032)	662	78.9	2002-2015
SHY	0.024*** (0.002)	662	78.9	2002-2015
LQD	0.121*** (0.015)	662	78.9	2002-2015
Aaa [†]	-2.666*** (0.406)	871	126.6	1995-2015
Panel B. Equities				
SPY	0.016 (0.027)	871	126.6	1995-2015
IWM	0.039 (0.060)	706	91.4	2000-2015
SP500 [†]	-0.111 (0.113)	871	126.6	1995-2015
Russell 2000 [†]	-0.191 (0.119)	871	126.6	1995-2015
Panel C. Inflation expectations and commodities				
10Y Inflation Swap [†]	-0.290 (0.331)	618	74.2	2004-2015
2Y Inflation Swap [†]	-0.001 (0.669)	618	74.2	2004-2015
GLD	0.041 (0.030)	595	72.3	2004-2015
GSCI [†]	0.023 (0.107)	871	126.6	1995-2015
Panel D. Spreads and credit default swaps				
Baa-Aaa [†]	-0.169 (0.146)	871	126.6	1995-2015
3-month LIBOR-OIS [†]	-0.006 (0.004)	630	77.3	2003-2015
Auto CDS [†]	-9.793 (7.458)	627	77.0	2004-2015
Bank CDS [†]	1.269 (0.800)	627	77.0	2004-2015
VIX [†]	-0.063 (0.148)	871	126.6	1995-2015

Notes: The table repeats the regressions from Table 4, but instruments $D_t^{(m')}$ with the surprise component of the bid-to-cover ratio. First-stage F-statistics are reported in column (3). Newey-West standard errors in parentheses.

Table B2: Secondary Market Rate Responses

Panel A: Short-Term (2-7 year) auctions						
	(1)	(2)	(3)	(4)	(5)	(6)
	0-2	2-5	5-8	8-11	11-20	20-30
$C_t=0 \times D_t$	-1.32***	-2.16***	-2.48***	-1.97***	-2.46***	-1.79***
	(0.22)	(0.30)	(0.33)	(0.30)	(0.29)	(0.24)
$C_t=1 \times D_t$	-1.44***	-2.61***	-3.09***	-3.02***	-2.72***	-2.09***
	(0.25)	(0.39)	(0.46)	(0.45)	(0.44)	(0.45)
Observations	35363	37304	15644	7813	9733	11085
Clusters	615	615	615	615	615	615
R^2	0.136	0.217	0.224	0.205	0.206	0.153
P-Value	0.712	0.366	0.274	0.053	0.625	0.561

Panel B: Long-Term (10-30 year) auctions						
	(1)	(2)	(3)	(4)	(5)	(6)
	0-2	2-5	5-8	8-11	11-20	20-30
$C_t=0 \times D_t$	-0.56***	-1.07***	-1.55***	-1.83***	-2.21***	-2.13***
	(0.20)	(0.30)	(0.40)	(0.33)	(0.39)	(0.26)
$C_t=1 \times D_t$	-1.04***	-1.96***	-2.49***	-2.95***	-3.00***	-3.15***
	(0.25)	(0.29)	(0.30)	(0.33)	(0.34)	(0.38)
Observations	15139	16525	7264	3385	4161	4559
Clusters	255	255	255	255	255	255
R^2	0.152	0.216	0.277	0.329	0.349	0.369
P-Value	0.144	0.032	0.064	0.017	0.136	0.028

Notes: The table reports results estimating equation (4), but using security-level changes in yields as the dependent variable. Panel A reports the results for short-term auction dates (2-7 years), while Panel B reports the results for long term auction dates (10-30 years). The columns break up the securities into different baskets based on the remaining maturity: column (1) contains all note and bonds with less than 2 years remaining before maturity; column (2) is 2-5 years; column, column (3) is 5-8 years; column (4) is 8-11 years; column (5) is 11-20 years; and column (6) is 20-30 years. P-values testing equality of coefficients are reported in the final row. Standard errors clustered at the auction level are in parentheses.

Table B3: Numerical Exercise Calibration

Parameter	Value
T	30
σ	.01
κ_r	0.7
κ_s	0.3
κ_ℓ	0.3
α	5
a	(0, 500)
$\theta_s(m)$	$\delta(m - 3)$
$\theta_\ell(m)$	$\delta(m - 20)$