The Twelfth Dubrovnik Economic Conference

Organized by the Croatian National Bank

HNB

Dirk Krueger and Alexander Ludwig

On the Consequences of Demographic Change for Rates of Returns to Capital, and the Distribution of Wealth and Welfare

Hotel "Grand Villa Argentina",
Dubrovnik
June 28 - July 1, 2006

Draft version

Please do not quote

On the Consequences of Demographic Change for Rates of Returns to Capital, and the Distribution of Wealth and Welfare*

Dirk Krueger
Goethe University Frankfurt, CEPR, CFS, MEA and NBER

Alexander Ludwig
University of Mannheim and MEA

April 11, 2006

Abstract

In the industrialized world the population is aging over time, reducing the fraction of the population in working age. Consequently labor is expected to be scarce, relative to capital, with an ensuing decline in the real return on capital. This paper uses demographic projections together with a large scale multi-country Overlapping Generations Model with uninsurable idiosyncratic uncertainty to quantify the distributional and welfare consequences of these changes in factor prices induced by the demographic transition.

In our model capital can freely flow between different regions in the OECD (the U.S., the EU and the rest of the OECD). Thus international capital flows may in principle mitigate the decline in rates of returns one would expect in the U.S. if it were a closed economy.

We find exactly the opposite. In the U.S. as an open economy, rates of return are predicted to decline by 86 basis points between 2005 and 2080. If the U.S. were a closed economy, this decline would amount to only 78 basis points. This result is due to the fact that other regions in the OECD will age even more rapidly; therefore the U.S. is "importing" the more severe aging problem from these regions, especially Europe. A similar conclusion is reached if we let capital flow freely between the OECD and the rest of the world (ROW). While ROW currently has a younger population structure, it is predicted to age even more severely in the next decades, giving rise to an even more pronounced decline in world rates of return to capital.

*We thank participants of seminars at the LSE, the Ente Einaudi, MEA and the 2005 Cleveland FED International Macroeconomics conference for many useful comments. The authors can be reached at dirk.krueger@wiwi.uni-frankfurt.de and alexander.ludwig@mea.uni-mannheim.de. This paper was prepared for the Spring 2006 Carnegie-Rochester Conference on Public Policy.

In order to evaluate the welfare consequences of the demographic transition we ask the following hypothetical question: suppose a household economically born in 2005 would live through the economic transition with changing factor prices induced by the demographic change (but keeping her own survival probabilities constant at their 2005 values), how would its welfare have changed, relative to a situation without a demographic transition? We find that households experience significant welfare losses due to the demographic transition, in the order of $2-5\%$ of consumption, depending on their initial productivity level and the design of the pension system. These losses are mainly due to the fact that lower future returns to capital make it harder for households to save for retirement. On the other hand, if the OECD suddenly opens up to ROW in 2005 and ROW has higher returns to capital before the world capital market integration, then these losses are reduced to $1.5-2.5\%$.

JEL Classification: E17, E25, D33, C68

Keywords: Population Aging, International Capital Flows, Distribution of Welfare

1 Introduction

In all major industrialized countries the population is aging, over time reducing the fraction of the population in working age. This process is driven by falling mortality rates followed by a decline in birth rates, which reduces population growth rates (and even turns it negative in some countries). While demographic change occurs in almost all countries across the world, extent and timing differ substantially. Europe and some Asian countries have almost passed the closing stages of the demographic transition process while Latin America and Africa are only at the beginning stages (Bloom and Williamson, 1998; United Nations, 2002).

Figure 1 (which is based on UN population projections) illustrates the differential impact of demographic change on population growth rates for the period 2000-2080 for four regions of the world that comprise the entire world and that we will use throughout this paper: the U.S., the European Union (EU), the rest of the OECD (ROECD) and the rest of the world (ROW). Population growth rates are predicted to decline in all regions, but are positive in the U.S. and in the ROW region throughout the 21st century, whereas they significantly fall below zero in the EU in about 2016 and in ROECD in about 2042. As a consequence, the population in the EU (the ROECD) starts shrinking in about 2016 (2042), whereas the population in the other two regions continues to increase.

Figure 2 shows the impact of demographic change on working-age population ratios - the ratios of the working-age population (of age 20-64) to the total population (of age 20-95). This indicator, which will turn out to be crucial in our analysis) illustrates that the EU is the oldest, whereas the ROW is the youngest region in terms of the relative size of the working-age population. The United States and the rest of the OECD region initially have the same level of working-age population ratios, but the dynamics of demographic change

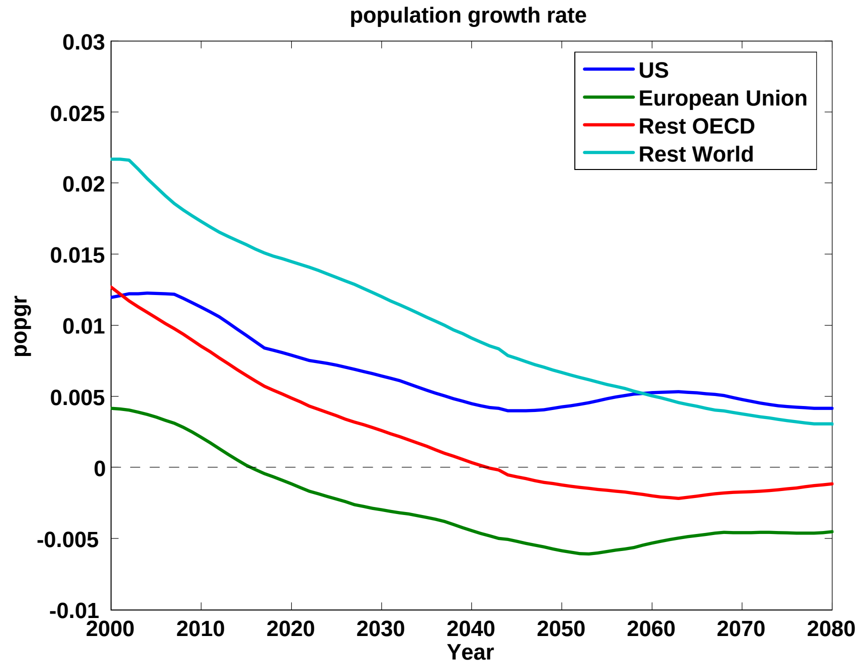

Figure 1: Evolution of the Population Growth Rate in 4 Regions

differ substantially in the U.S. relative to the other regions. While working-age population ratios decrease across all regions, the speed of this decrease significantly slows down for the U.S. in about 2030.

What are the welfare consequences of living in a world where the population is aging rapidly? First, individuals live longer lives and tend to have fewer children, which are the underlying reasons of aging populations. The welfare effects of these changes are hard to quantify. Second, due to changes in the population structure, aggregate labor supply and aggregate savings is bound to change, with ensuing changes in factor prices for labor and capital. Specifically, labor is expected to be scarce, relative to capital, with an ensuing decline in the real return on capital. The primary objective of this paper is to quantify the distributional and welfare consequences from this second, general equilibrium effect of the demographic changes around the world.

To this end, we use demographic projections from the United Nations, together with a large scale Overlapping Generations Model pioneered by Auerbach and Kotlikoff (1987). We extend the model to a multi-country version, as in Börsch-Supan et al. (2005) and many others, and also enrich the model by uninsurable idiosyncratic uncertainty, as in Imrohoroglu et al. (1995), Imrohoroglu et al. (1999), Conesa and Krueger (1999) and many others. Both extensions of the basic Auerbach-Kotlikoff model are necessary for the question we want

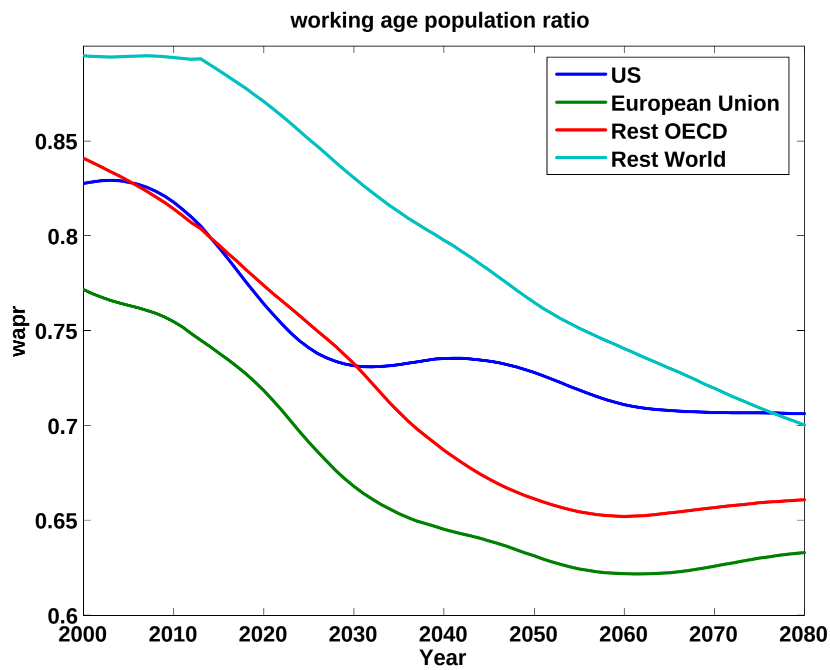

Figure 2: Evolution of Working Age to Population Ratios in 4 Regions

to address. First, uninsurable idiosyncratic uncertainty will endogenously give rise to some agents deriving most of their income from returns to capital, while the income of others is mainly composed of labor income. Abstracting from this heterogeneity does not allow a meaningful analysis of the distributional consequences of changes in factor prices. This feature also adds a precautionary savings motive to the standard life-cycle savings motive of households, which makes life cycle profiles of consumption generated by the model more realistic. Second, in light of potential differences in the evolution of the age distribution of households across regions, it is important to allow for capital to flow across regions. In our model capital can freely flow between different regions in the OECD (the U.S., the EU and the rest of the OECD). Thus international capital flows may in principle mitigate the decline in rates of return one would expect in the U.S. if it were a closed economy.

We find exactly the opposite. In the U.S. as an open economy, rates of return are predicted to decline by 86 basis points between 2005 and 2080. If the U.S. were a closed economy, this decline would amount to only 78 basis points. This result is due to the fact that other regions in the OECD will age even more rapidly; therefore the U.S. is "importing" the more severe aging problem from these regions, especially Europe. A similar conclusion is reached if we let capital flow freely between the OECD and the rest of the world (ROW). While ROW currently has a younger population structure, it is predicted to age even more severely in the next decades (see again figure 2), giving rise to an even more pronounced decline in world rates of return to capital.

In order to evaluate the welfare consequences of the demographic transition we ask the following hypothetical question: suppose a household economically born in 2005 would live through the economic transition with changing factor prices induced by the demographic change (but keeping her own survival probabilities constant at their 2005 values), how would its welfare have changed, relative to a situation without a demographic transition? We find that households experience significant welfare losses due to the demographic transition, in the order of $2-5\%$ of consumption, depending on their initial productivity level and on the design of social security systems. These losses are mainly due to the fact that lower future returns to capital make it harder for households to save for retirement. On the other hand, if the OECD suddenly opens up to ROW in 2005 and ROW has higher returns to capital before the world capital market integration, then these losses are reduced to $1.5-2.5\%$.

Our paper borrows model elements from, and contributes to, three strands of the literature. Starting with Auerbach and Kotlikoff (1987) a vast number of papers has developed that uses large-scale OLG models to analyze the transition path of an economy induced by a policy reform. Examples include social security reform (see e.g. Conesa and Krueger (1999), fundamental tax reform (see e.g. Altig et al. (2001), Conesa and Krueger (2005)) and many others.

A second strand of the literature (often using the general methodology of the first strand) has focused on the economic consequences of population aging in closed economies, often paying special attention to the adjustments required in the social security system due to demographic shifts. Important examples include Huang et al. (1997), De Nardi et al. (1999), and with respect to asset prices, Abel (2003).

These contributions discussed so far assume that the economy under investigation is closed to international capital flows. However, as the population ages at different pace in various regions of the world one would expect capital to flow across these regions. The third strand of the literature our paper touches upon therefore is the large body of work in international macroeconomics studying the direction, size, cause and consequences of international capital flows and current account dynamics in a variety of models. This large literature is reviewed comprehensively in Obstfeld and Rogoff (1995).

Our paper is most closely related to work that combines these three strands of the literature, by using the methodology of large scale OLG models to study the consequences of demographic change in open economies. The work by Attanasio et al. (2006b) constructs a two region (the North and the South) OLG model to study the allocative and welfare consequences of different social security reforms in an open economy. Compared to their model, we include endogenous labor supply and idiosyncratic income shocks. While we also have to take a stand on how the social security system deals with the aging of the population, these social security reforms are not in the center of our analysis whereas their paper focuses on this issue. In Attanasio et al. (2006a) the authors quantify the direct welfare losses from demographic changes for the South region of their model, carrying out a similar thought experiment we do for the U.S. Their qualitative and quantitative results are consistent with the findings of our paper.

Similar to Attanasio et al. (2006b), but with a stronger focus on Europe or the OECD, Börsch-Supan et al. (2005) and Fehr et al. (2005) investigate the impact of population aging on the viability of the social security system and its reform. Building on earlier work by Brooks (2003) who employs a simple four period OLG model, Henriksen (2002), Feroli (2003) and Domeij and Floden (2005) use large scale simulation models similar to Attanasio et al. (2006a, b) to explain historical capital flow data with changes in demographics, rather than, as we do, to study the (welfare and distributional) implications of future changes in demographics. Relative to this literature, we see the contribution of our paper in evaluating the welfare consequences of the demographic transition (with focus on the U.S.) per se (and not just the alternative social security reform scenarios), as well as in the analysis of the distributional consequences of changing factor prices due to population aging.

The paper is organized as follows. In the next section we construct a simple, analytically tractable multi-country OLG model to isolate the key determinants of the direction of international capital flows and the impact of changes in the demographic structure on rates of return and capital flows. Section 3 contains the description of our large scale simulation model, including the definition of competitive equilibrium. Section 4 discusses the calibration of the model and section 5 presents results for the benchmark model. In section 6 we document the sensitivity of our results with respect to our assumptions about the regions between which capital can flow freely, and we also show that ignoring endogenous labor supply responses to factor price changes may result in biased predictions of the model. Section 7 concludes, and separate appendices contain more detailed information about the demographic model underlying our simulations, as well as details of the computational strategy and calibration of the model.

2 A Simple Model

In this section we construct a simple OLG model that is a special case of our quantitative model in the next section. We can characterize equilibria in this model analytically, and are especially interested in the influence of demographic variables and the size and structure of the social security system on rates of return to capital and the direction and dynamics of international capital flows. The results of the simple model along these dimensions will provide some intuition for the quantitative results from the simulation model.

In every country i there are $N_{t,i}$ young households that live for two periods and have preferences over consumption $c^y_{t,i}, c^o_{t+1,i}$ representable by the utility function

$$\log(c^y_{t,i}) + \beta \log(c^o_{t+1,i})$$

In the first period of their lives households work for a wage $w_{t,i}$, and in the second period they retire and receive social security benefits $b_{t+1,i}$ that are financed via payroll taxes on labor income. Thus the budget constraints read as

$$\begin{aligned} c_t^y + s_t &= (1-\tau_{t,i})w_{t,i} \\ c_{t+1}^o &= (1+r_{t+1})s_t + b_{t+1,i} \end{aligned}$$

where r_{t+1} is the real interest rate between period t and $t+1$ and $\tau_{t,i}$ is the social security tax rate in country i. We assume that capital flows freely across countries, and thus the real interest rate is equalized across the world.

The production function in each country is given by

$$Y_{t,i} = K_{t,i}^{\alpha} \left(Z_i A_t N_{t,i}\right)^{1-\alpha},$$

where Z_i is the country-specific technology level and $A_t = (1+g)^t$ is exogenously growing productivity. Thus we allow for differences in technology levels across countries, but not its growth rate. We further assume that capital depreciates fully after use in production.

The production technology in each country is operated by a representative firm that behaves competitively in product and factor markets. Profit maximization of firms therefore implies that

$$\begin{aligned} 1+r_t &= \alpha k_t^{\alpha-1} \\ w_{t,i} &= (1-\alpha)Z_i A_t k_t^{\alpha}, \end{aligned} \tag{1}$$

where

$$k_t = k_{t,i} = \frac{K_{t,i}}{Z_i A_t N_{t,i}} \ \forall i$$

is the capital stock per efficiency unit of labor.

We assume that the social security system is a pure pay-as-you-go (PAYGO) system that balances the budget in every period. Therefore

$$\tau_{t,i} w_{t,i} N_{t,i} = b_{t,i} N_{t-1,i}$$

Finally, market clearing in the world capital market requires that

$$K_{t+1} = \sum_i K_{t+1,i} = \sum_i N_{t,i} s_{t,i}$$

2.1 Analysis

Equilibria in this model can be characterized analytically. For that purpose we first solve the household problem and then aggregate across households (countries).

2.1.1 Optimal Household Savings Behavior

From the household problem we can solve for saving of the young in country i as

$$s_{t,i} = \frac{\beta}{1+\beta} w_{t,i}(1-\tau_{t,i}) - \frac{b_{t+1,i}}{(1+\beta)(1+r_{t+1})} \tag{2}$$

The budget constraint of the social security system implies that

$$b_{t,i} = \frac{N_{t,i}}{N_{t-1,i}} w_t \tau_{t,i} = \gamma_{t,i}^N w_{t,i} \tau_{t,i}$$

where $\gamma_{t,i}^N$ is the gross growth rate of the young cohort in country i between period $t-1$ and t. It also measures the working age to population ratio (the higher is $\gamma_{t,i}^N$, the higher is that ratio) in this model[1], which allows us to map the predictions of this model back to the data, plotted in figure 2. Using this expression for benefits and substituting out for wages and interest rates from (1) in (2) yields

$$s_{t,i} = \frac{\beta(1-\tau_{t,i})(1-\alpha)}{1+\beta} Z_i A_t k_t^{\alpha} - \frac{\gamma_{t+1,i}^N \tau_{t+1,i}(1-\alpha)}{(1+\beta)\alpha} Z_i A_{t+1} k_{t+1} \tag{3}$$

2.1.2 Aggregation

For further reference, define by $\tilde{N}_t = \sum_i Z_i N_{t,i}$ the efficiency weighted world population, by $\tilde{\theta}_{t,i} = \frac{Z_i N_{t,i}}{\tilde{N}_t} = \frac{\tilde{N}_{t,i}}{\tilde{N}_t}$ the relative share of the efficiency-weighted population in country i and by $\tilde{\gamma}_t^N = \frac{\tilde{N}_t}{\tilde{N}_{t-1}} = \sum_i \tilde{\theta}_{t,i} \gamma_{t,i}^N$ the growth rate of aggregate (world) efficiency weighted population.

The capital market clearing condition reads

$$\sum_i s_{t,i} N_{t,i} = \sum_i K_{t+1,i} = k_{t+1} \sum_i Z_i A_{t+1} N_{t+1,i} = k_{t+1} A_{t+1} \tilde{N}_{t+1} \tag{4}$$

[1] The population of a country i at time t is given by

$$Pop_{t,i} = N_{t,i} + N_{t-1,i}$$

and the working age to population ratio is given by

$$wapr_{t,i} = \frac{N_{t,i}}{Pop_{t,i}}.$$

The we can easily compute the growth rate of the population as

$$\gamma_{t,i}^{Pop} = \frac{Pop_{t+1,i}}{Pop_{t,i}} = \frac{1+\gamma_{t,i}^N}{1+\frac{1}{\gamma_{t-1,i}^N}}$$

In a steady state

$$\gamma_i^{Pop} = \gamma_i^N$$

Also $wapr_{t,i} = \frac{1}{1+\frac{1}{\gamma_{t,i}^N}}$. Thus $\gamma_{t,i}^N$ measures both the population growth rate as well as the working age to population ratio.

Aggregating household savings decisions across countries yields, from (3):

$$\sum_i s_{t,i}N_{t,i} = \frac{(1-\alpha)\beta A_t k_t^\alpha}{1+\beta}\sum_i (1-\tau_{t,i})Z_i N_{t,i} - \frac{(1-\alpha)A_{t+1}k_{t+1}}{(1+\beta)\alpha}\sum_i Z_i N_{t+1,i}\tau_{t+1,i}$$

Using this in (4) and simplifying yields

$$k_{t+1} = \sigma(\tilde{\gamma}^N_{t+1}, \gamma^A, \tau^a_t, \tau^a_{t+1})k_t^\alpha \tag{5}$$

where

$$\begin{aligned}\sigma_t &= \sigma(\tilde{\gamma}^N_{t+1}, \gamma^A, \tau^a_t, \tau^a_{t+1}) = \\ &= \frac{\alpha(1-\alpha)\beta(1-\tau^a_t)}{\tilde{\gamma}^N_{t+1}\gamma^A\left(\alpha(1+\beta)+(1-\alpha)\tau^a_{t+1}\right)}\end{aligned}$$

is the world aggregate saving rate of the economy in period t, with $\tau^a_t = \sum_i \tau_{t,i}\tilde{\theta}_{t,i}$ denoting the average social security contribution rate in the world and $\gamma^A = 1+g$ is the growth rate of the technology.

Equation (5), as a function of the policy and demographic parameters of the model, describes the dynamics of the aggregate capital stock, given the initial condition k_0.[2] As long as $\tilde{\gamma}^N_{t+1}\gamma^A \geq 1$, the economy converges monotonically from its initial condition to a balanced growth path in which all per capita variables grow at the rate g of technical progress.[3]

Since, from the firms' first order condition, interest rates are given by

$$1 + r_t = \alpha k_t^{\alpha-1}$$

the dynamics of the real interest rate are given by

$$1 + r_{t+1} = \left(\frac{\alpha}{\sigma_t}\right)^{1-\alpha}(1+r_t)^\alpha \tag{6}$$

with initial condition $1 + r_0 = \alpha k_0^{\alpha-1}$.

Finally, we can characterize international capital flows. Net foreign assets of country i at the beginning of period $t+1$ (or the end of period t) are given by

$$F_{t+1,i} = N_{t,i}s_{t,i} - K_{t+1,i} = N_{t,i}s_{t,i} - Z_i A_{t+1}N_{t+1,i}k_{t+1}$$

Thus, after some tedious algebra, and using (3) yields

$$\begin{aligned}\frac{F_{t+1,i}}{Y_{t,i}} &= \left(\frac{(1-\alpha)\beta(1-\tau_{t,i})}{1+\beta} - \gamma^N_{t+1,i}\gamma^A\sigma_t\left(1+\frac{\tau_{t+1,i}(1-\alpha)}{(1+\beta)\alpha}\right)\right) \\ &= f_{t+1,i}(\gamma^N_{t+1,i}, \gamma^A, \tau_{t,i}, \tau_{t+1,i}, \tilde{\gamma}_{N,t+1}, \tau^a_t, \tau^a_{t+1})\end{aligned} \tag{7}$$

as the net foreign asset to GDP ratio of country i.[4]

[2] Explicitly, $k_0 = \frac{\sum_i s_{-1,i}N_{-1,i}}{A_0\sum_i Z_i N_{0,i}}$ where $s_{-1,i}N_{-1,i}$ denotes total assets held by the initial old generation in country i.

[3] This is a sufficient, but by no means necessary condition.

[4] Furthermore, the current account, relative to output, is defined as

$$ca_{t,i} = \frac{CA_{t,i}}{Y_{t,i}} = \frac{F_{t+1,i}-F_{t,i}}{Y_{t,i}} = f_{t+1,i} - f_{t,i}/\gamma^Y_{t,i} \tag{8}$$

2.2 Qualitative Results

In this section we illustrate how changes in demographics, or changes in the social security system (induced by demographic changes) affect both world-wide rates of return and net foreign asset positions of different countries.

2.2.1 Dynamics in Rates of Return

What are the effects of an (unexpected, but permanent) decline in the working age to population ratio in period t? Since k_t and hence r_t is predetermined, we observe from (6) that the response of r_{t+1} depends negatively on the world saving rate σ_t, which is itself a negative function of the efficiency-weighted population growth rate $\tilde{\gamma}_{t+1}^N$ between period t and $t+1$ (which can also be interpreted as a measure of the worldwide working age to population ratio). Evidently a decline in $\tilde{\gamma}_{t+1}^N$ increases the world-wide saving rate σ_t, which in turn reduces the rate of return r_{t+1} tomorrow.[5]

Another interesting, and possibly somewhat unexpected observation from (6) pertains to the potential indirect consequences from population aging via the social security system. The aggregate saving rate σ_t depends negatively on the size of the social security system, as measured by its (world average) contribution rate τ^a. If the population ages and if policy makers want to keep social security *benefits* stable, this requires an increase in contribution rates. But such an increase, according to our simple model reduces σ_t and thus *increases* future rates of return. This effect may mitigate or even dominate the direct effect of population aging on rates of return, as also highlighted by Fehr et al. (2005).

To summarize, a decline in the world-wide working age to population ratio (as approximated by $\tilde{\gamma}^N$) leads to a decline in rates of return to capital, as long as social security contribution rates are held constant (and thus benefits shrink). If, however, contribution rates are raised in addition, to keep social security benefits stable, the predicted decline in returns is smaller, or returns may even increase. Quantitative work is needed to measure the relative strength of these effects, something we will turn to in the next sections of this paper.[6]

where $\gamma_{t,i}^Y = \frac{Y_{t,i}}{Y_{t-1,i}} = \gamma_t^A \gamma_{t,i}^N \left(\sigma_{t-1} k_{t-1}^{\alpha-1}\right)^\alpha$ is the growth rate of output in country i.

[5] The future dynamics of rates of return can then be read off from (6), with σ_τ being constant for $\tau > t$ and returns converging monotonically to the lower steady state level associated with the lower $\tilde{\gamma}^N$.

[6] For the case that $\tau_{t,i} = \tau_{t+1,i} = 0$ one can also derive a simple expression for the welfare consequences of a decline in $\tilde{\gamma}_{t+1}^N$. In that case lifetime utility of an agent born in t is given by

$$u_t = \kappa + (1+\beta)\log(w_{t,i}) + \beta \log(1 + r_{t+1}).$$

In response to a decline in $\tilde{\gamma}_{t+1}^N$ wages $w_{t,i}$ do not change (since it depends k_t which is predetermined at time t), but reduces $1 + r_{t+1}$, leading to an unambiguous welfare loss for generation t.

For future generations the welfare consequences depend on the relative magnitudes of declines in rates of return and increases in wages. One can show that the lower is α, the stronger is the response of interest rates, relative to wages, and thus the more unfavorable are the welfare consequences from a decline in $\tilde{\gamma}^N$. On the other hand, the higher is β, the more

2.2.2 Net Foreign Asset Positions

Finally we want to deduce the implications of the simple model for the sign and dynamics of net foreign asset positions across countries. While it is possible to derive these implications in general, it is cleanest and most instructive for intuition to focus on balanced growth paths in which the economy is growing at rate $\gamma^N \gamma^A$ and where the capital stock per efficiency unit of labor is given as

$$k^* = (\sigma^*)^{\frac{1}{1-\alpha}} = \left(\frac{\alpha(1-\alpha)\beta(1-\tau^a)}{\tilde{\gamma}^N \gamma^A \left(\alpha(1+\beta) + (1-\alpha)\tau^a\right)} \right)^{\frac{1}{1-\alpha}}$$

Evidently the steady state capital stock per labor efficiency units is strictly decreasing in the effective population growth rate of the world, $\tilde{\gamma}^N$ as well as the average social security contribution rate of the world economy, τ^a. The reverse is true for the world interest rate.

Along the balanced growth path, net foreign asset positions of country i are given by[7]

$$f_i = \frac{\beta(1-\alpha)(1-\tau_i)}{1+\beta} \left[1 - \frac{\tilde{\gamma}_i^N (1-\tau^a)(\alpha(1+\beta) + (1-\alpha)\tau_i)}{\tilde{\gamma}_N (1-\tau_i)(\alpha(1+\beta) + (1-\alpha)\tau^a)} \right]$$

Thus our simple model has the following qualitative predictions for net asset positions.[8] First, if all countries have identical population growth rates and social security contribution rates $\tilde{\gamma}_i^N = \tilde{\gamma}^N$ and $\tau_i = \tau^a$), then net asset positions and current accounts are zero in the long run. Second, if all countries have the same size of the social security system ($\tau_i = \tau^a$), then

$$f_i = \frac{\beta(1-\alpha)(1-\tau_i)}{1+\beta} \left[1 - \frac{\tilde{\gamma}_i^N}{\tilde{\gamma}^N} \right]$$

Therefore countries with higher than world average working age to population ratios have a negative net asset position and countries with lower than world average working age to population ratios have positive net asset positions. Capital flows from old to young regions. While the steady state assumption makes it tricky to discuss the dynamics of net asset positions within the model, we would still expect that countries whose working age to population ratios decline faster (slower) than others experience an increase (decline) in the net foreign asset position.[9]

---

important are interest rate changes for welfare, and thus the higher β, the more unfavorable are the welfare effects from a decline in $\tilde{\gamma}^N$.

[7] We made use of the fact that $\tilde{N}_{t,i} = Z_i N_{t,i}$ and thus $\tilde{\gamma}_i^N = \gamma_i^N$

[8] In the empirically relevant case that $\tilde{\gamma}_i^N \gamma^A > 1$, the sign of the current account coincides with that of the net foreign asset position, since the former is given by

$$ca_i = f_i \left(1 - \left(\tilde{\gamma}_i^N \gamma^A \right)^{-1} \right)$$

[9] Strictly speaking, an increase in $\tilde{\gamma}_i^N$ or τ_i changes τ^a as well. This is meant by ceteris paribus. Also note that, with country heterogeneity in $\tilde{\gamma}_i^N$, in the long run, almost the entire world population is concentrated in the country with the highest $\tilde{\gamma}_i^N$, but that per-capita variables are still well-defined in all other countries as well.

Finally, if all countries have identical working age to population ratios ($\tilde{\gamma}_i^N = \tilde{\gamma}^N$) then

$$f_i = \frac{\beta(1-\alpha)(1-\tau_i)}{1+\beta}\left[1 - \frac{(1-\tau^a)(\alpha(1+\beta)+(1-\alpha)\tau_i)}{(1-\tau_i)(\alpha(1+\beta)+(1-\alpha)\tau^a)}\right]$$

and countries with higher than average social security contribution rates, $\tau_i > \tau^a$ have negative net asset positions and those with lower contribution rates have positive net asset positions and current accounts.

We will again use these qualitative predictions from the simple model to interpret our results from the quantitative model to which we turn next.

3 The Quantitative Model

In this section we describe the quantitative model we use to evaluate the consequences of demographic changes around the world for international capital flows, for the returns to capital and wages, as well as the welfare consequences emanating from these changes. In our quantitative work we consider (at most) four countries/regions: the United States (U.S.), the European Union (EU), the rest of the OECD (ROECD) and the rest of the world (ROW).

3.1 Demographics

The demographic evolution in the countries of interest is taken as exogenous (that is, we do not model fertility, mortality or migration endogenously) and is the main driving force of our model. Households start their economic life at age 20 retire at age 65 and life at most until age 95. Since we do not model the first 19 years of a household explicitly, we denote its twentieth year of life by $j = 0$ its retirement age by $jr = 45$ and the terminal age of life by $J = 75$. Households face an idiosyncratic, time- and country-dependent (conditional) probability to survive from age j to age $j+1$ which we denote by $s_{t,j,i}$.

For each country $i \in \{1, \ldots, I\}$ we have data or forecasts for populations of model age $j \in \{0, \ldots, 75\}$ in years $1950, \ldots, 2300$. In the remainder of the paper, we denote year 1950 as our base year $t = 0$ and year 2300 as the final period T and the demographic data for periods $t \in \{0, \ldots, T\}$ by $N_{t,j,i}$. For simplicity, we assume that all migration takes place at or before age $j = 0$ in the model, so that we can treat migrants and agents born inside the country of interest symmetrically. The law of motion of the demographic data for age $j \geq 0$ is accordingly given by

$$N_{t+1,j+1,i} = s_{t,j,i} N_{t,j,i}$$

3.2 Technology

In each country the single consumption good is being produced according to a standard neoclassical production function

$$Y_{t,i} = Z_i K_{t,i}^{\alpha} \left(A_t L_{t,i}\right)^{1-\alpha}$$

where $Y_{t,i}$ is output in country i at date t, $K_{t,i}$ and $L_{t,i}$ are labor and capital inputs and A_t is total labor productivity, growing at a constant rate, g, which is the same across countries. Z_i denotes total factor productivity in country i which scales countries by their average levels of productivity. The parameter α measures the capital share and is assumed to be constant over time and across countries. Furthermore, in each country, capital used in production depreciates at a rate δ, again assumed to be time- and country-independent. Since production takes place with a constant-returns to scale production function in each country and since we assume perfect competition, the number of firms is indeterminate in equilibrium and, without loss of generality, we assume that a single representative firm operates within each country.

3.3 Endowments and Preferences

Households value consumption and leisure over the life cycle $\{c_j, 1-l_j\}$ according to a standard time-separable utility function

$$E\left\{\sum_{j=0}^{J}\beta^j u(c_j, 1-l_j)\right\},$$

where β is the raw time discount factor and expectations are taken over idiosyncratic mortality shocks and stochastic labor productivity.

Households are heterogenous with respect to age, a deterministic earnings potential and stochastic labor productivity. All these sources of heterogeneity affect a household's labor productivity and thus wages. First, households labor productivity differs according to their age. Let ε_j denote average age-specific productivity of cohort j. Second, each household belongs to a particular group $k \in \{1, \ldots, K\}$ that shares the same average productivity θ_k. Differences in groups stand in for differences in education or ability, characteristics that are fixed at entry into the labor market and affect a group's relative wage. We introduce these differences in order to generate part of the cross-sectional income and thus wealth dispersion that does not come from our last source of heterogeneity, idiosyncratic productivity shocks. That is, lastly, a household's labor productivity is affected by an idiosyncratic shock, $\eta \in \{1, \ldots, E\}$ that follows a time-invariant Markov chain with transition probabilities

$$\pi(\eta'|\eta) > 0.$$

Let Π denote the unique invariant distribution associated with π. Therefore, labor productivity of a household of age j in group k and with idiosyncratic shock η is given by

$$\theta_k \varepsilon_j \eta.$$

3.4 Government Policies

The government collects assets of households that die before age J and redistributes them in a lump-sum fashion among the citizens of the country as accidental bequests, $h_{t,i}$ (in**h**eritances). Furthermore, we explore how our results are affected by the presence and the design of a pure pay-as-you-go public pension system, whose taxes and benefits have to be adjusted to the demographic changes in each country. This social security system is modelled as follows. On the revenue side, households pay a flat payroll tax rate $\tau_{t,i}$ on their labor earnings. Retired households receive benefits, $b_{t,k,i}$, that are assumed to depend on the household type, θ_k, but are independent of the history of idiosyncratic productivity shocks. Pension benefits are therefore given by

$$b_{t,k,i} = \rho_{t,i}\theta_k(1 - \tau_{t,i})w_{t,i},$$

where $\rho_{t,i}$ is the pension system's net replacement rate.

We assume that the budget of the pension system is balanced at all times such that taxes and benefits are related via the government budget constraint by

$$\tau_{t,i}w_{t,i}L_{t,i} = \sum_k b_{t,k,i} \sum_{j \leq jr} N_{t,j,k,i}, \tag{9}$$

where $N_{t,j,k,i}$ denotes the size of the population at time t of age j and type k in country i.

In our results section we distinguish between two different scenarios for the future evolution of the social security system in different countries, one in which taxes are held constant and replacement rates adjust accordingly, and vice versa. The results from the simple model above suggests that our results will be significantly affected by this modelling choice.

3.5 Market Structure

In each period there are spot markets for the consumption good, for labor and for capital services. While the labor market is a national market where labor demand and labor supply are equalized country by country, the markets for the consumption good and capital services are international in that goods and capital can flow freely, and without any transaction costs, between countries. The supply of capital stems from households in all countries who purchase capital as assets in order to save for retirement and to smooth out idiosyncratic productivity shocks. The supply of consumption goods stems from the representative firms in each country.

Again, as sensitivity analysis, we explore how the U.S. would be affected by its demographic changes if it were a closed economy. In that exercise the capital used by U.S. firms equals the assets that U.S. citizens accumulate for life cycle and precautionary motives.

3.6 Equilibrium

Individual households, at the beginning of period t are indexed by their age j, their group k, their country of origin i, their idiosyncratic productivity chock η, and their asset holdings a. Thus their maximization problem reads as

$$\begin{aligned} & W(t,j,k,i,\eta,a) \\ = & \max_{c,a',1-l}\{u(c,1-l)+\beta s_{t,j,i}\sum_{\eta'}\pi(\eta'|\eta)W(t+1,j+1,k,i,\eta',a')\} \\ & \text{s.t. } c+a' = \begin{cases}(1-\tau_{t,i})w_{t,i}\theta_k\varepsilon_j\eta l+(1+r_t)(a+h_{t,i}) & \text{for } j<jr \\ b_{t,k,i}+(1+r_t)(a+h_{t,i}) & \text{for } j\geq jr\end{cases} \\ & a',c\geq 0 \text{ and } l\in[0,1] \end{aligned} \tag{10}$$

Here $w_{t,i}$ is the wage rate per efficiency unit of labor and r_t is the real interest rate. We denote the cross-sectional measure of households in country i at time t by $\Phi_{t,i}$. We can then define a competitive equilibrium as follows.

Definition 1 *Given initial capital stocks and distributions,* $\{K_{0,i},\Phi_{0,i}\}_{i\in I}$*, a competitive equilibrium are sequences of individual functions for the household,* $\{W(t,\cdot),c(t,\cdot),l(t,\cdot),a'(t,\cdot)\}_{t=0}^{\infty}$*, sequences of production plans for firms* $\{L_{t,i},K_{t,i}\}_{t=0,i\in I}^{\infty}$*, policies* $\{\tau_{t,i},\rho_{t,i},b_{t,i}\}_{t=0,i\in I}^{\infty}$*, prices* $\{w_{t,i},r_t\}_{t=0,i\in I}^{\infty}$*, transfers* $\{h_{t,i}\}_{t=0,i\in I}^{\infty}$ *and measures* $\{\Phi_{t,i}\}_{t=0,i\in I}^{\infty}$ *such that*

1. *Given prices, transfers and initial conditions,* $W(t,\cdot)$ *solves equation* (12)*, and* $c(t,\cdot),l(t,\cdot),a'(t,\cdot)$ *are the associated policy functions.*

2. *Interest rates and wages satisfy*

$$\begin{aligned} r_t &= \alpha Z_i\left(\frac{K_{t,i}}{A_tL_{t,i}}\right)^{\alpha-1}-\delta \\ w_{t,i} &= (1-\alpha)Z_iA_t\left(\frac{K_{t,i}}{A_tL_{t,i}}\right)^{\alpha} \end{aligned}$$

3. *Transfers are given by*

$$h_{t+1,i}=\frac{\int(1-s_{t,j,i})a'(t,j,k,i,\eta,a)\Phi_{t,i}(dj\times dk\times d\eta\times da)}{\int\Phi_{t+1,i}(dj\times dk\times d\eta\times da)} \tag{11}$$

4. *Government policies satisfy* (3.4) *and* (9) *in every period*

5. *Market clearing*

$$\begin{aligned} & L_{t,i}=\int\theta_k\varepsilon_j\eta l(t,j,k,i,\eta,a)\Phi_{t,i}(dj\times dk\times d\eta\times da) \text{ for all } i \\ & \sum_{i=1}^{I}K_{t+1,i}=\sum_{i=1}^{I}\int a'(t,j,k,i,\eta,a)\Phi_{t,i}(dj\times dk\times d\eta\times da) \\ & \sum_{i=1}^{I}\int c(t,j,k,i,\eta,a)\Phi_{t,i}(dj\times dk\times d\eta\times da)+\sum_{i=1}^{I}K_{t+1,i} \\ & =\sum_{i=1}^{I}A_{t,i}K_{t,i}^{\alpha}L_{t,i}^{1-\alpha}+(1-\delta)\sum_{i=1}^{I}K_{t,i} \end{aligned}$$

6. *Law of Motion for cross-sectional measures* Φ*: The cross-sectional measures evolve as*

$$\Phi_{t+1,i}(\mathcal{J}\times\mathcal{K}\times\mathcal{E}\times\mathcal{A}) = \int P_{t,i}((j,k,\eta,a),\mathcal{J}\times\mathcal{K}\times\mathcal{E}\times\mathcal{A})\Phi_{t,i}(dj\times dk\times d\eta\times da)$$

where the Markov transition functions $P_{t,i}$ *are given by*

$$P_{t,i}((j,k,\eta,a),\mathcal{J}\times\mathcal{K}\times\mathcal{E}\times\mathcal{A}) = \begin{cases} \pi(\eta,\mathcal{E})s_{t,j,i} & \textit{if } a'(t,j,k,i,\eta,a)\in\mathcal{A} \\ & k\in\mathcal{K}, j+1\in\mathcal{J} \\ 0 & \textit{else} \end{cases}$$

and for newborns

$$\Phi_{t+1,i}(\{1\}\times\mathcal{K}\times\mathcal{E}\times\mathcal{A}) = N_{t+1,0,i}\cdot\begin{cases} \Pi(\mathcal{E}) & \textit{if } 0\in\mathcal{A} \\ 0 & \textit{else} \end{cases}$$

Definition 2 *A stationary equilibrium is a competitive equilibrium in which all individual functions are constant over time and all aggregate variables grow at a constant rate.*

3.7 Thought Experiment and Computation

We use our model for the following thought experiment. We take as exogenous driving process a time-varying demographic structure in all regions under consideration. We allow country specific survival, fertility and migration rates to change over time, inducing a demographic transition from an initial distribution towards a final steady state population distribution that arises once all time changes in these rates have been completed and the population structure has settled down to its new steady state. Induced by this transition of the population structure is a transition path of the economies of the model, both in terms of aggregate variables as well as cross-sectional distributions of wealth and welfare. Summary measures of these changes will provide us with answers as to how the changes in the demographic structure of the economy, by changing returns to capital and wages, impact the distribution of welfare over time and across people in the economy.

The time line of our model is therefore as follows: we start computations in year 1950 assuming an artificial initial steady state. We then use data for a calibration period, 1950-2004, to determine several structural model parameters, see section 4. We then report simulation results for our projection period from 2005-2080. However, we also solve the model for a phase-out period that lasts until 2300 when a final steady state is reached. For given structural model parameters we solve for the equilibrium using a modification of the familiar Gauss-Seidel algorithm (Ludwig, 2005a). Throughout we take as length of the period one year. Appendix B contains a detailed description on how we solve the household model for given prices and how we solve for aggregate equilibrium quantities and prices over $T = 350$ years.

4 Calibration

In this section we discuss how we specify the parameters for our benchmark model. This entails choosing parameters governing the demographic transition, the production technology by firms, the endowment and preference specification of households and the social security policy.

4.1 Demographics

Our demographic data is based on the United Nations world population projections (United Nations, 2001). These population numbers determine both the idiosyncratic survival probabilities as well as the relative sizes of total populations in the four countries/regions in all time periods under consideration. Figures 1 and 2 in the introduction summarized the main stylized facts from these population figures, and appendix A describes in detail the nature of our demographic data and the methodology underlying our demographic projections.

4.2 Technology

With respect to the parameters of the production functions, we restrict the capital share parameter, α, the growth rate of labor productivity, g, and the depreciation rate, δ, to be constant across the four regions under consideration, whereas we allow technology levels Z_i to differ across regions. The parameters of the production technologies in the different countries can accordingly be collected as

$$\vec{\Psi}^{PS} = [\alpha, g, \delta, Z_1, \ldots, Z_4]' \,.$$

We estimate parameters α, g and δ using U.S. NIPA data for a sample period of 1950-2004, set $Z_1 = 1$ and estimate $Z_2, \ldots, Z_4$ taking data on relative labor productivity across regions. A more detailed description of our approach is given in appendix B.3. Table I summarizes the resulting parameter estimates for the full model with four world regions.

Table I: Technology Parameters

Parameter	U.S.	EU	ROECD	ROW
Capital Share α	0.33			
Growth Rate of Technology g	0.018			
Depreciation Rate δ	0.04			
Total Factor Productivity Z_i	1.0	0.88	0.65	0.11

4.3 Endowments and Preferences

Households start their life with no assets and are endowed with one unit of time per period. Labor productivity is given by the product of three components, a deterministic age component ε_j, a deterministic group component θ_k and a stochastic idiosyncratic component η.

The age-productivity profile $\{\varepsilon_j\}_{j=1}^J$ is taken from Hansen (1993) and generates an average life-cycle wage profile consistent with (U.S.) data. Conditional on age, the natural logarithm of wages is given by

$$\log(\theta_k) + \log(\eta).$$

We choose the number of groups to be $K = 2$ and let groups be of equal size. We choose $\{\theta_1, \theta_2\}$ such that average-group productivity is equal to 1 and the variance of implied labor incomes of entrants to the labor market coincides with that reported by Storesletten et al. (2004). This requires $\theta_1 = 0.57$ and $\theta_2 = 1.43$. For the idiosyncratic part of labor productivity we use a 2 state Markov chain with annual persistence $\rho = 0.98$ and implied conditional variance of 8%.

We assume that the within period utility function is of the familiar CRRA form given by

$$u(c, l) = \frac{1}{1-\sigma} \left(c^{\omega_i}(1-l)^{1-\omega_i}\right)^{1-\sigma},$$

where σ denotes the coefficient of relative risk aversion and where ω_i measures the relative importance of consumption, relative to leisure in each country. Differences in ω_i across countries allow us to match simulated hours worked to the actual data separately for each country. In addition we have to specify the time discount factor of households which we restrict to be identical across countries.

The parameters of the household sector can accordingly be summarized as

$$\vec{\Psi}^{HS} = [\sigma, \beta, \omega_1, \ldots, \omega_4]'.$$

We assume $\sigma = 1$ such that utility is separable between consumption and leisure, and determine the value of the discount rate by matching the average simulated capital-output ratio to U.S. data for the period 1950-2004. The consumption share parameters ω_i are estimated by matching simulated average hours worked in the regions of our model to the data. A more detailed description of our methodology is given in appendix B.3. Table II summarizes the parameters of the household model for the version of our model where all regions are included, labor supply is endogenous and where pension systems are present.[10]

Table II: Preference Parameters

Parameter	U.S.	EU	ROECD	ROW
Coefficient of RRA σ	1.0			
Time Discount Factor β	0.9422			
Consumption Share Parameter ω_i	0.459	0.446	0.444	0.500

[10] Estimated parameter values for other alternative versions of our model used in the sensitivity analysis are similar and available from the authors upon request. For each alternative model all model parameters are recalibrated to match the same aggregate data described above.

4.4 Social Security System

Our benchmark model contains no social security system. The version of the model used most prominently in our welfare calculations contains the PAYGO social security system, uses historical data for social security tax rates in the four regions of interest until 2004 and then freezes future contribution rates at their 2004 levels. Benefits adjust to achieve budget balance. In the alternative scenario of fixed replacement rates we again use historical region-specific data on contribution rates to back out constructed replacement rates until 2004 and then fix replacement rates in the future to their 2004 values. Tax rates adjust (i.e. increase) to assure budget balance of the social security system.

Data for calibrating the social security system are taken from various sources. For the U.S., we calculate social security contribution rates from NIPA data taken from the BEA (Table 3.6). It is more difficult to obtain data for the other world regions. We proxy the time path of social security contribution rates by using time path information on total labor costs taken from the BLS and scale these data by social security contribution rates from the OECD observed for one year.

5 Results for the Benchmark Model

In order to isolate the direct effects of demographic changes on returns to capital, international capital flows, and the distribution of wealth and welfare we first abstract from social security. In the presence of such a system demographic changes necessitate reforms in either social security taxes or benefits, which induce further changes on aggregate variables and distributions. We will quantify these effects in section 5.4. In the benchmark scenario we also assume that capital flows freely only between regions in the OECD, and we document in section 6.1 how our results are affected if capital flows to the rest of the world are permitted.

5.1 Aggregate Statistics

In figure 3 we display the evolution of the real return to capital from 2000 to 2080. In the same figure we plot, as a summary measure of the age structure of the population, the fraction of the world population with age above 65 (by assumption these agents are retired in our model); this statistic is one minus the working age to population ratio. We observe that the rate of world-wide return to capital is predicted to fall by about 1 percentage point in the next 60 years and then to settle down at that lower level. This is exactly what we would have expected, given the qualitative results from the simple model in section 2, and given the fact that so far we abstract from social security (reform).

Given our production function, pre-tax wages are related to the interest rate by

$$w_{t,i} = (1-\alpha) Z_i A_t \left(\frac{\alpha Z_i}{r_t + \delta} \right)^{\frac{\alpha}{1-\alpha}}$$

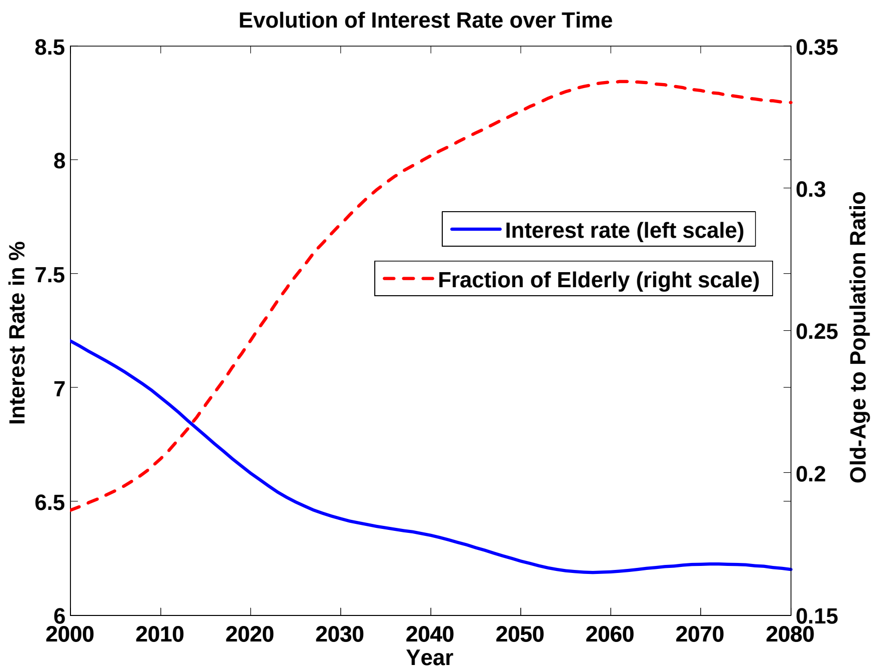

Figure 3: Evolution of World Interest Rates

and thus de-trended (by productivity growth) real wages follow exactly the inverse path of interest rates, as documented in figure 3. These de-trended wages are predicted to increase by roughly 5% between 2000 and 2080 in all regions in our model.

In figure 4 we plot the evolution of de-trended output per capita in the three regions, normalized to 1 in the year 2000. We observe substantial declines of $7-13\%$ in the three regions. The decline is least pronounced in the U.S., since there the decrease of the fraction of households in working age is more modest after 2030, as we saw in figure 2.

5.2 Quantifying International Capital Flows

In order to document our results about the direction and size of international capital flows we will document the evolution of the net asset position and the current account of the countries/regions under consideration. Define the net foreign asset position of country i at time t at the beginning of period t as

$$F_{t,i} = A_{t,i} - K_{t,i}$$

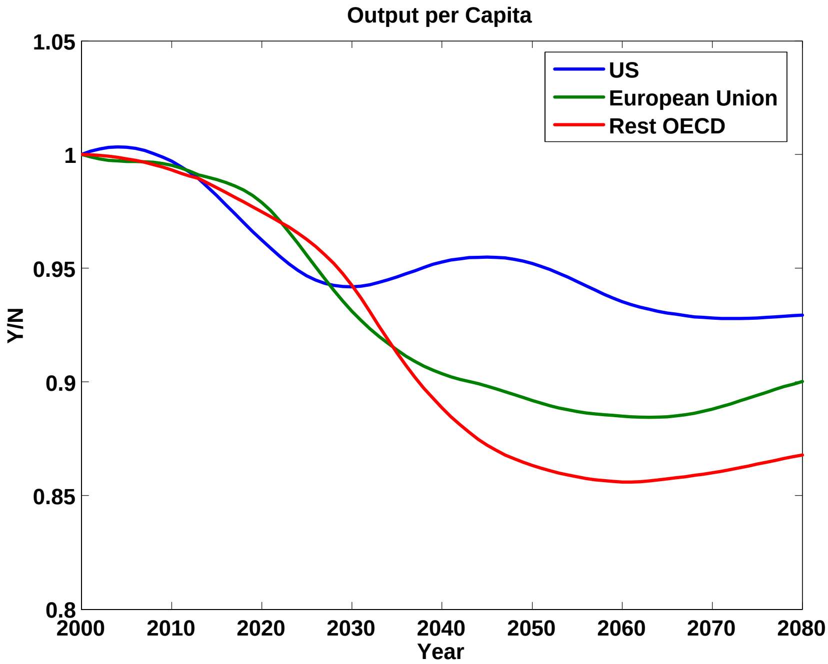

Figure 4: Evolution of GDP per Capita in 3 Regions

The current account in period t is then defined as the change in the net asset position of a country,

$$\begin{aligned} CA_{t,i} &= F_{t+1,i} - F_{t,i} \\ &= (A_{t+1,i} - A_{t,i}) - (K_{t+1,i} - K_{t,i}) \\ &= S_{t,i} - I_{t,i} \end{aligned}$$

that is, the current account equals the difference between national savings and domestic investment of a country.[11] When reporting these statistics we always divide them by output $Y_{t,i}$.

We first plot, in figure 5, the net foreign asset position, relative to GDP, in the three regions of our model. The European Union, as the oldest region, has a positive net asset position and thus provides capital to both the rest of the OECD and, increasingly, to the U.S., whose population is aging slowest.

[11] Note that in a closed economy $F_{t,i} = C_{t,i} = 0$, and that in a balanced growth path of an open economy $CA_{t,i} = g\,(A_{t,i} - K_{t,i})$. Furthermore net asset positions and current accounts evidently have to sum to 0 across regions:

$$\sum_i F_{t,i} = \sum_i CA_{t,i} = 0 \text{ for all } t.$$

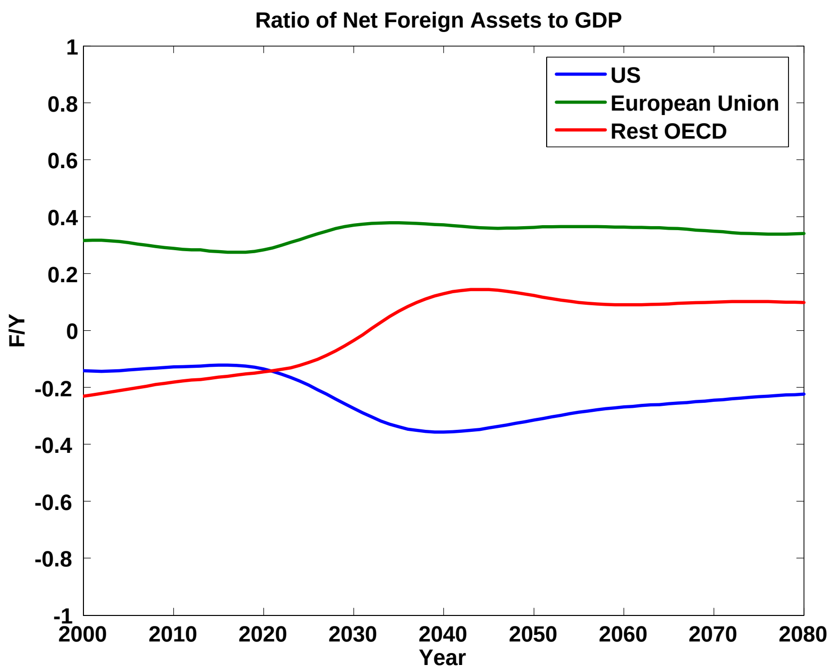

Figure 5: Evolution of the Net Foreign Asset Position in 3 Regions

especially in the years between 2020 and 2035 our model predicts massive capital inflows into the U.S. and out of European Union and, especially, the rest of the OECD. Again this is exactly in line with the predictions of the simple model, coupled with the level and dynamics of the working age to population ratio in these different regions, as documented in figure 2.

The current account of a country is simply the change in the net asset position, and thus the derivative of the previous plots. Figure 6 shows very clearly the deterioration of the current account of the U.S. that is expected to occur in the next 30 years, as capital flows from the European Union and, with a slight time delay, from the rest of the OECD, into the U.S. By 2040 this process is completed and the current account of all countries returns to roughly 0 from that point on.

A negative current account such as predicted for the U.S. in the next 40 years can be due to weak saving or strong investment in a country. In order to disentangle the different effects of demographic changes on the current account, in figures 7 and 8 we plot the evolution of region-specific saving rates $\frac{S_{t,i}}{Y_{t,i}}$ and investment rates $\frac{I_{t,i}}{Y_{t,i}}$ over time.

The most direct effect of an aging population is that labor, as a factor of production, becomes scarce. As a result, for unchanged aggregate saving the

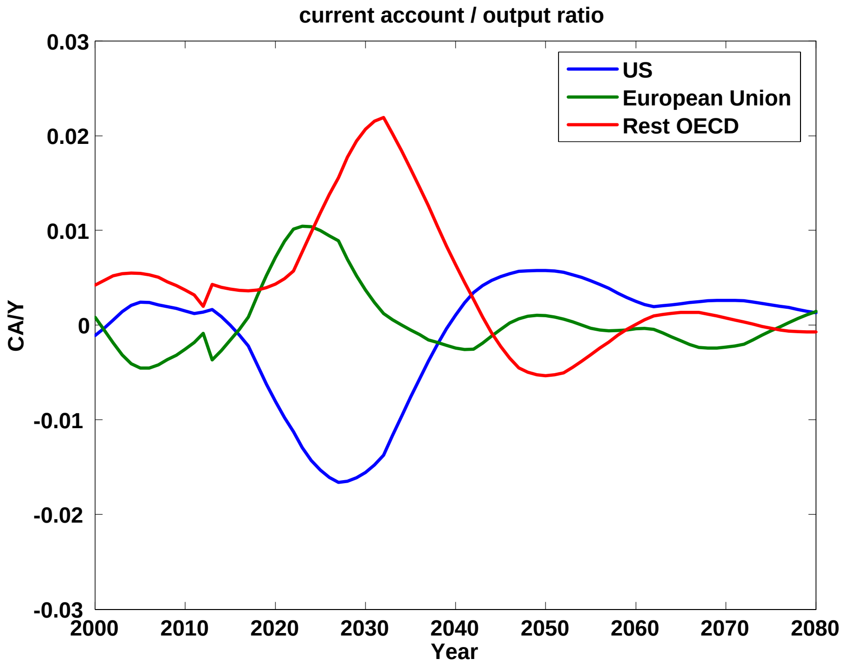

Figure 6: Evolution of the Current Account in 3 Regions

return to capital has to fall and gross wages have to rise. This is what we observe in figure 3. However, the decline in interest rates reduces the incentives of households to save. In addition, with the aging of society the age composition of the population shifts towards older households, who are dis-savers in our life cycle model. Consequently savings rates in all regions in our model decline over time, as shown in figure 7. For the next 20 years the fall in savings rates is most pronounced for the U.S., because there, during this time period, the large cohort of baby boomers moves into retirement. The same is true for other regions of the world, albeit to a lesser degree.

After the large cohort of baby boomers have left the economy (i.e. died) the U.S. saving rate is predicted to rebound (in about 25 to 35 years) and then to stabilize, whereas in the European Union and the rest of the OECD savings rate continue to fall until about 2040 and then stabilize.

The other side of the medal (that is, of the current account) is the investment behavior in the different regions. Given that savings rates decline globally due to population aging investment rates have to do so as well (at least on average), since the world current account has to balance to 0. Figure 8 demonstrates this fall for all regions, but also shows that the fall is by far the least pronounced for the U.S. Furthermore, in the U.S. the investment rate stops to fall by about 2020, roughly a decade earlier than its saving rate. This is due to the fact that

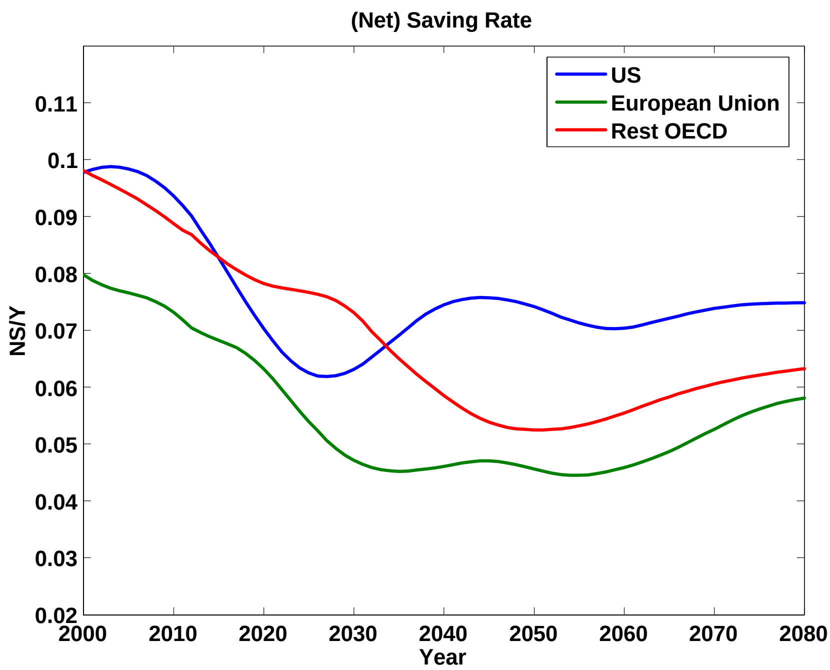

Figure 7: Evolution of Net Saving Saving Rate in 3 Regions

the fall in the working age to population ratio is more or less completed around that date in the U.S. On the other hand, in the EU and the rest of the OECD this ratio continues to fall until 2035. Since capital-(effective) labor ratios have to be equalized, capital allocated to these regions has to fall (relative to the U.S.) and so do investment rates in these regions. As a consequence we observe the large deterioration of the current account in the U.S. of about 2% of GDP around 2025.

5.3 Distributional and Welfare Consequences of Demographic Change

In the previous sections we have documented substantial changes in factor prices induced by the aging of the population, amounting to a decline of about 1 percentage point in real returns to capital and an increase in gross wages of about 5% in the next decades. In this section we want to quantify the distributional and welfare effects emanating from these changes.

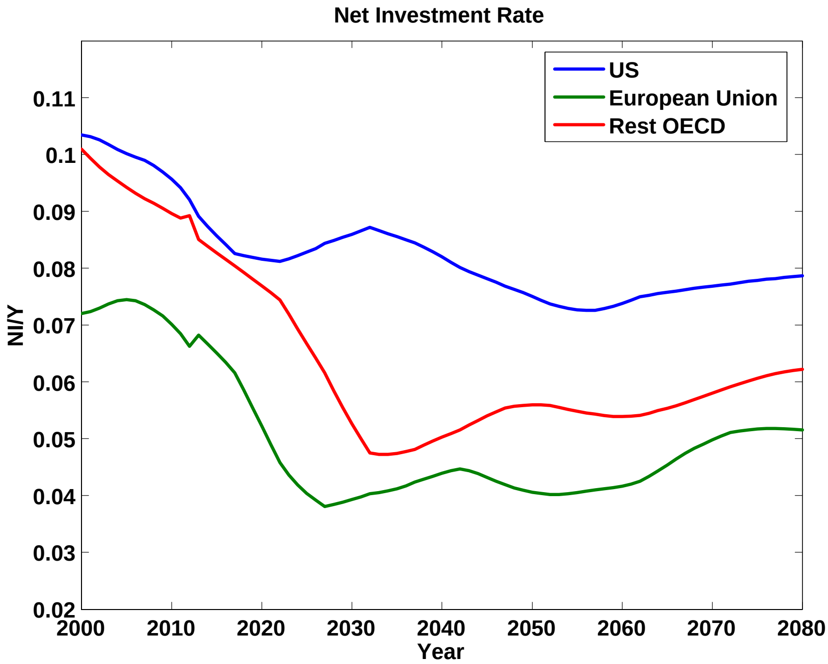

Figure 8: Evolution of Net Investment Rate in 3 Regions

5.3.1 Evolution of Inequality

In figure 9 we display the evolution of income inequality over time in the three regions. Income is composed of labor income (which later will include pension income) and capital income as well as transfers from accidental bequests. We observe a significant increase in income inequality between 2000 and 2080, of about 5 points in the Gini coefficient for the EU and the ROECD and 3.5 points in the U.S.. The reason for this increase is mainly a compositional effect. Retired households have significantly lower income on average than households in working age. The demographic transition towards more retired households therefore is bound to increase inequality, especially in those regions where the increase in the fraction of retired households among the population is very pronounced. This explains the more modest increase in income inequality in the U.S.. Note that consumption inequality follows income inequality trends fairly closely in the three regions (and thus is not shown here), but increases in consumption inequality are less pronounced. Also notice that the ordering of countries in the figure will be reversed once we add pension systems - then, income will be least equally distributed in the U.S..

The fact that it is not a rise in capital income inequality that drives the increase in total income inequality becomes clear when plotting wealth inequality

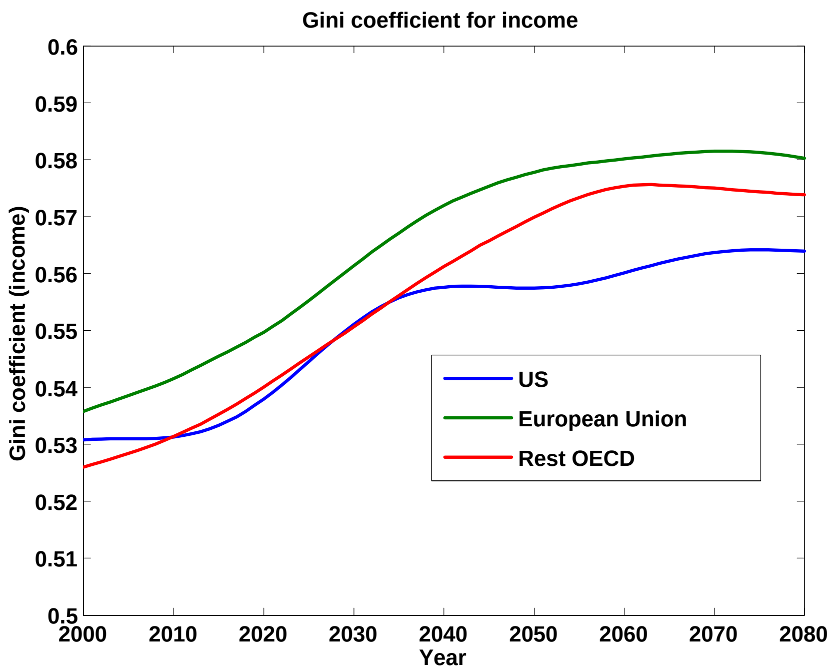

Figure 9: Evolution of Income Inequality in 3 Regions

over time. There is no discernible increase in the same period; evidently the same is true for capital income inequality since capital income is proportional to wealth.

In contrast to income, wealth follows a hump-shaped pattern over the life cycle (on average), with the elderly and the young being wealth-poor. Thus, in contrast to income inequality, the aging of the population does not lead to an increase in wealth inequality, since the demographic change increases the fraction of the elderly, but reduces the fraction of the young. Consequently income and wealth inequality do not follow the same trend over time (nor is the ranking in inequality across regions the same for income and wealth).

5.3.2 Welfare Consequences of the Demographic Transition

A household's welfare is affected by two consequences of the demographic change. First, her lifetime utility changes because her own survival probabilities increase; this is in part what triggers the aging of the population (the other source are declines in birth rates). Second, due to the demographic transition she faces different factor prices and government transfers and taxes (from the social security system and from accidental bequests) than without changes in the demographic structure.

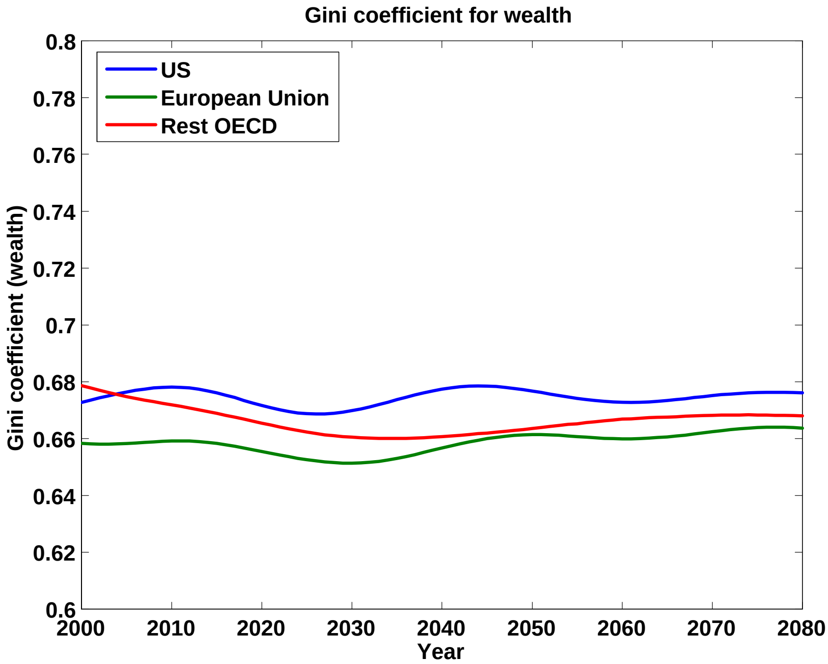

Figure 10: Evolution of Wealth Inequality in 3 Regions

We want to isolate the welfare consequences of the second effect. For this we compare lifetime utility of agents born and already alive in 2005 under two different scenarios. For both scenarios we fix a household's individual survival probabilities at their 2005 values; of course they fully retain their age-dependence. Then we solve each household's problem under two different assumptions about factor prices (and later taxes/transfers, once we have introduced social security). Let $\overline{W}(t,i,j,k,\eta,a)$ denote the lifetime utility of an agent at time $t \geq 2005$ in country i with individual characteristics (j,k,η,a) that faces the sequence of equilibrium prices as documented in the previous section, but constant 2005 survival probabilities, and let $\overline{W}_{2005}(t,i,j,k,\eta,a)$ denote the lifetime utility of the same agent that faces prices and taxes/transfers that are held constant at their 2005 value. Finally, denote by $g(t,i,j,k,\eta,a)$ the percentage increase in consumption that needs to be given to an agent (t,i,j,k,η,a) at each date and contingency in her remaining lifetime (keeping labor supply allocations fixed) at fixed prices to make her as well off as under the situation with changing prices.[12]

[12] For the Cobb-Douglas utility specification for $\sigma \neq 1$ the number $g(t,i,j,k,\eta,a)$ can easily be computed as

$$g(t,i,j,k,\eta,a) = \left[\frac{\overline{W}(t,i,j,k,\eta,a)}{\overline{W}_{2005}(t,i,j,k,\eta,a)}\right]^{\frac{1}{\omega_i(1-\sigma)}}.$$

Negative numbers of $g(t, i, j, k, \eta, a)$ thus indicate that households suffer welfare losses from the general equilibrium effects of the demographic changes.[13] Of particular interest are the numbers $g(t = 2005, i, j = 1, k, \eta, a = 0)$, that is, the welfare consequences for newborn agents in 2005 (remember that newborns start their life with zero assets).

Table III documents the welfare consequences for *newborns* in all three regions, differentiated by their type k and productivity shock η. We make several observations. First, independent of country of origin, productivity type and initial productivity shock newborn agents experience welfare losses from changing factor prices and transfers induced by the demographic transition. Apart from changing preferences through higher longevity (an effect we control for in our welfare calculations) the demographic transition substantially reduces the interest rate, moderately increases wages and somewhat reduces transfers from accidental bequests, at least eventually. The negative first and third effect dominate the second effect of higher wages, especially the need to save for retirement at a lower interest rate, in the absence of social security, adversely affects welfare.[14]

Table III: Welfare Consequences, Newborns

	United States		Eur. Union		Rest OECD	
	η_1	η_2	η_1	η_2	η_1	η_2
k_1	−5.3%	−2.0%	−5.4%	−2.1%	−5.0%	−1.8%
k_2	−5.2%	−1.9%	−5.3%	−2.0%	−4.9%	−1.7%

Second, the welfare losses differ across household characteristics and countries. In the benchmark model without social security, cross-country differences in welfare losses can only be due to country-specific changes in transfers from accidental bequests, with more severe declines being associated with higher welfare losses. Since transfers decline by 23% in the EU, but only by 20% and 12% in the U.S. and the rest of the OECD between 2005 and 2080, the welfare losses are highest in the EU, followed by the U.S. and then ROECD. Since these transfers are small in magnitude, however, so are the cross-country differences. Absent social security, lump-sum transfers from accidental bequests are also the only source of nonhomotheticity in the model. Since these transfers are less important for high-productivity agents, their decline hurts them less and thus the welfare losses are moderately smaller for k_2 types than for k_1 types. Again the difference is small since transfers are.

Quantitatively more important are differences in welfare losses across stochastic productivity shocks, which are significantly higher for agents starting the labor market with low productivity. These agents are not permanently less

A similar expression holds for $\sigma = 1$.

[13] We also computed these numbers taking 1950 as the base year of comparison. The results are available upon request.

[14] If interest rates are held constant, the welfare consequences are indeed (significantly) positive for all but the wealth-poorest who suffer from the decline in transfers.

productive than their brothers, but given the persistent nature of the shocks they will have to wait a while in expectation before becoming productive and thus in a position to save for retirement. But this means that they have to save for retirement at lower interest rates, since these are falling over time. Currently productive agents, on the other hand, can take advantage of their currently high labor income and start to accumulate assets at times in which returns are still high. In addition to this effect, the eventual reduction in transfers again hurts income-rich agents less than income-poor agents. Combining these effects results in welfare losses of currently poor young households that are substantially higher.

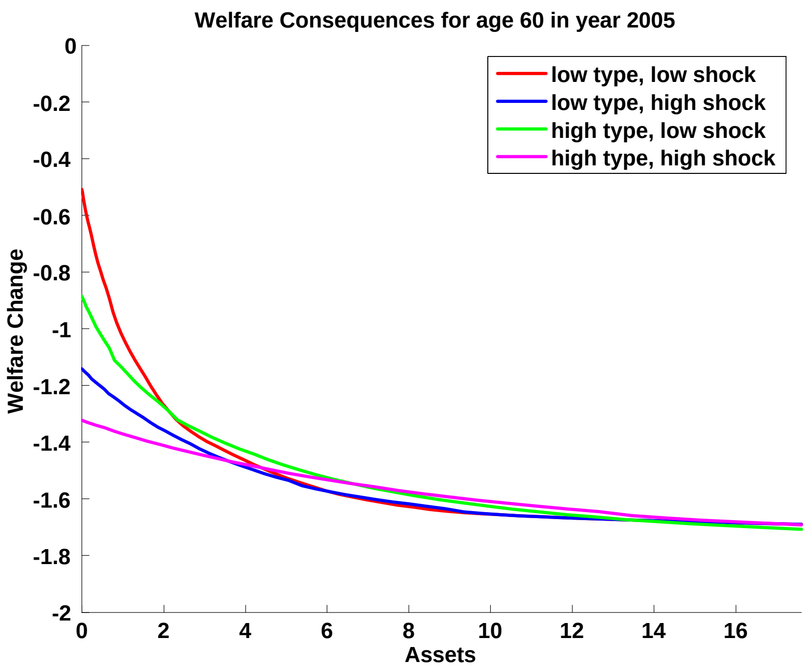

Figure 11: Welfare Consequences by Asset Levels

An advantage of our model with uninsurable idiosyncratic income shocks and thus intra-cohort heterogeneity is that it allows us to document how the welfare losses are distributed across the population, both across and within cohorts. To demonstrate this by example, figure 11 plots the welfare losses for agents of age 60 in 2005. These households have most of their working life behind them, thus are fairly unaffected by the wage changes, and simply experience lower returns on their accumulated savings. While all households of that age group suffer welfare losses, these losses differ substantially by accumulated asset levels, with asset-rich households losing significantly more than asset-poor households.

5.4 The Role of Social Security (and its Reform)

So far, we completely abstracted from government policies. While it is not clear a priori what the interaction of demographic change and public policies are in general, it is abundantly clear from policy debates that at least one large social program is strongly affected by it: social security.

An idealized pay as you go public pension system can respond to an increase in the share of pensioners in the population by (a combination of) two ways: cutting benefits or increasing social security contribution rates. While a likely response will include both elements, we now present results for the model with a PAYGO social security system that responds to population aging by either holding tax rates fixed (and thus cutting benefits), or by holding replacement rates fixed (and thus raising taxes). Because of the strong influence of a public pension system on private savings behavior, we expect that these different reform scenarios may have substantially different implications for the evolution of factor prices and the size and direction of international capital flows as well as the distribution of wealth and welfare. This conjecture turns out to be correct. Note that for all exercises we re-calibrate production and preference parameters such that each economy (with the different social security systems) attains the same calibration targets for the 1950 to 2004 period

In table IV we show how the evolution of macroeconomic aggregates and prices differs across different reform scenarios for social security. On the other hand, keeping pension benefits constant has dramatic consequences for the evolution of interest rates and wages, relative to the benchmark scenario of fixing tax rates for social security. With fixed benefits the incentives to save for retirement are drastically reduced, relative to the benchmark. In addition, the substantial increase in tax rates and thus reduction in after tax wages makes it harder to save. Therefore, despite the decline in the fraction of households in working age (and diminished incentives to work because of higher payroll taxes) now the capital-labor ratio remains roughly unchanged, because of the large reduction of household savings. De-trended per capita GDP therefore declines much more sharply under this reform scenario than under the benchmark of fixed contribution rates.

Table IV. Evolution of Aggregates, 2005-2080[15]

Var.	No Soc.Sec.	τ fixed	ρ fixed
r	-0.89%	-0.86%	-0.17%
w	4.4%	4.1%	0.8%
$(1-\tau)w$	4.4%	4.1%	$-7.3\%, -11.8\%, -15.4\%$
Y/N	$-7.35\%, -9.7\%, -13.1\%$	$-7.1\%, -9.2\%, -12.6\%$	$-12.6\%, -14.5\%, -18.0\%$

Given these substantial differences it is not surprising that the welfare consequences differ across these two scenarios as well. Table V summarizes the welfare

[15] The three numbers in each column of the last row refer to the three regions, the U.S., the EU and the ROECD.

losses from the demographic transition for newborns in the U.S. in 2005. The main observation from the table is that, in contrast to fixed contribution rates (may they be zero or positive) with fixed social security benefits welfare losses are much more evenly shared by households with different productivity shocks.[16] Social security, because of its progressive benefit schedule where benefits are imperfectly linked to contributions, redistributes from agents with currently high productivity (also remember that the process is highly persistent) to households with currently low productivity.

Table V: Welfare Consequences, Newborns in U.S.						
	No Soc.Sec.		τ Fixed		ρ Fixed	
	η_1	η_2	η_1	η_2	η_1	η_2
k_1	−5.3%	−2.0%	−4.5%	−1.8%	−2.6%	−2.5%
k_2	−5.2%	−1.9%	−4.5%	−2.0%	−2.6%	−2.6%

With fixed replacement rates and thus strongly increasing tax rates the magnitude of this redistribution increases, which reduces the adverse welfare effects from changing factor prices for η_1 households and magnifies them for η_2 households, relative to scenarios in which contribution rates and thus the size of the social security system remain unchanged.

6 Sensitivity Analysis

In this section we further investigate the driving forces behind our results. In particular, we discuss how our results hinge on our assumption of the U.S. being open to international capital flows from (parts of) the rest of the world, and we quantify the importance of modelling endogenous labor supply. Throughout these exercises, pension systems are included.

6.1 The Importance of Free International Capital Flows

6.1.1 The U.S. as Closed Economy

Most analyses of the demographic transition and their allocative consequences were carried out in a closed economy. We therefore want to quantify the importance of international capital flows for the extent to which the return to capital in the U.S. responds to a change in its demographic structure. In order to do so we now show results derived under the assumption that the U.S. is a closed economy, and thus the capital stock used in its production has to be equal to total domestic asset accumulation. For an interpretation of theses results it is

[16] Relative to a world without social security, now it is high productivity types that lose slightly more from demographic shifts, since accidental bequests initially increase in the presence of social security. This effect is quantitatively small, however.

important to keep in mind that, as figure 2 showed, if anything, the U.S. ages more slowly than the EU and the rest of the OECD.

Table VI: Evolution of Aggregates, 2005-2080		
Var.	Open Economy	Closed Economy
r	-0.86%	-0.78%
w	4.1%	3.7%
$(1-\tau)w$	4.1%	3.7%
Y/N	-7.1%	-7.7%

Table VI shows that the U.S. imports the more pronounced population aging from EU and rest of the world, in the sense that it experiences a more severe decline in rates of return in the U.S. as open economy, compared to as closed economy. Evidently wages follow the reverse pattern. Output per capita falls less in the open economy because both labor input and capital used in U.S. production decline by less between 2005 and 2080 than in the closed economy. For capital, another way of stating this fact is that in the closed economy the net foreign asset position of the U.S. is constant over time (and equal to zero), whereas in the open economy it is declining over time, as more capital is flowing into the U.S. (or less savings out of it) in 2080 than in 2005.

Measured by the decline in the return to capital, the U.S. therefore imports some of the adverse effects of more pronounced aging of the population in Europe and the rest of the OECD. Is this reflected in more negative welfare implications as well? Table VII demonstrates that indeed not only does the U.S. import negative consequences of population aging with respect to its rate of return, but also with respect to the welfare of its households living through a time where factor prices are changing due to changes in the demographic structure around the world. Quantitatively, however, the additional losses from faster aging societies elsewhere in the world are modest, in the order of about 0.3% of lifetime consumption. Europe and the rest of the OECD is aging faster than the U.S., but not to a vastly different degree.

Table VII: Welfare Consequences, Newborns in U.S.				
	Open Economy		Closed Economy	
	η_1	η_2	η_1	η_2
k_1	-4.5%	-1.8%	-4.2%	-1.5%
k_2	-4.5%	-2.0%	-4.3%	-1.6%

6.1.2 Opening Up to the Rest of the World

So far we have assumed that while the U.S. is an open economy relative to industrialized countries, capital is not flowing to developing countries, in our model the set of countries not part of the OECD. While this is an assumption that does not describe the real world exactly, the lack of capital flows from rich to poor countries has occupied such an influential literature that we took it as our point of departure.

Figure 2 demonstrates that even though the rest of the world is younger than OECD countries, it is expected to age faster in the next decades. Thus the intuition that adding the rest of the world to our calculations will mitigate the decline in the rate of return is flawed. In fact, repeating our thought experiment with four world regions (the fourth region being the rest of the world, ROW) delivers a decline in interest rates of about 1.04 percentage points (as compared to 0.86 percentage points in our benchmark) and somewhat higher welfare losses.[17]

These results do not imply, however, that there are no potential gains from international capital flows when dealing with aging populations. We demonstrate this with the following thought experiment: suppose until 2005 no capital flowed between the OECD and the rest of the world, and then, in 2006, barriers to international capital flows are suddenly (and unexpectedly) removed. Further, suppose that the rate of return on capital is initially higher in ROW.[18] Now capital flows from the OECD into ROW, making it scarcer in the OECD and thus mitigates, at least initially, the decline in rates of returns in the OECD.

Table VIII quantifies the magnitude of these beneficial effects from opening up the rest of the world to capital flows from the three regions of the OECD. It is most informative to display the changes in prices between 2004 and 2005, and between 2004 and 2030, since thereafter the changes in factor prices are nearly identical across the two scenarios (the three region economy and the opening-up scenario).

Table VIII. Evolution of Aggregates and Welfare (2005-80)

Variable	3 Region Scenario	Opening Up Scenario
$\Delta r_{2006-2005}$	-0.01%	0.14%
$\Delta r_{2030-2005}$	-0.62%	-0.47%
$\Delta w_{2006-2005}$	0.06%	-0.64%
$\Delta w_{2030-2005}$	2.86%	2.71%
$EV(k_1)$	$-4.5\%, -1.8\%$	$-2.3\%, -1.3\%$
$EV(k_2)$	$-4.5\%, -2.0\%$	$-2.5\%, -1.6\%$

[17] The fourth region is aging fast, rapidly reducing the share of the population in the labor force. In addition, it has only a small social security system, so that private savings are crucial for retirement consumption. An increasing share of elderly in this region makes for fairly stable national saving, further increasing the capital-labor share in the world economy.

[18] Specifically, we proceed as follows: we first calibrate the three region economy exactly as before. Now we assume that ROW has the same discount factor as the OECD regions and compute the equilibrium for ROW as a closed economy, up until 2005, under the assumption that all agents in the economy believe that ROW is closed to international capital flows forever. Since the rest of the world is a younger region than the OECD, the capital-labor ratio is lower there than in the OECD, and consequently rates of return to capital are higher. Then, in 2006, and unexpected for all agents the world opens up fully to international capital flows.

We observe that indeed rates of return are noticeably higher in the OECD after having opened up to the rest of the world, relative to remaining closed, for at least 30 years. The welfare losses (of newborns) from the demographic transition are substantially mitigated in the U.S., especially for those agents that would have suffered the most from interest rate declines in the near future (the η_1 households, the first welfare number in each row). The reduction in the welfare losses of 0.5% to 2%, depending on the group, may be interpreted as welfare gains from opening up the OECD to international capital flows with the rest of the world.

6.2 The Role of Endogenous Labor Supply

With the exceptions of Börsch-Supan et al. (2005) and Fehr et al. (2005), previous studies on the impact of demographic changes on rates of returns and international capital flows have abstracted from endogenous labor supply responses to changing factor prices. As the aging of the population tends to make labor scarce, and thus tends to drive up wages, it is likely that households choose to work more, reducing the effect of demographic changes on factor prices somewhat. In this subsection we want to quantify the importance of endogenous labor supply responses for our results.

Table IX shows that, as expected, abstracting from endogenous labor supply overstates the decline in real returns and output per capita to a substantial degree.

Table IX. Evolution of Aggregates, 2005-2080

Var.	Endog. Lab. Supply	Exog. Lab. Supply
r	-0.86%	-1.09
w	4.1%	5.4%
$(1-\tau)w$	4.1%	5.4%
Y/N	$-7.1\%, -9.2\%, -12.6\%$	$-10.2\%, -14.2\%, -15.9\%$

Consequently welfare losses from the demographic transition are more pronounced if households are not permitted to adjust labor supply, as shown in table X. This is the case especially for agents with currently high productivity shock, $\eta = \eta_2$. With endogenous labor supply, these agents take advantage of rising wages and work more, while this is not optimal to do for η_1 households. Thus welfare losses get reduced more strongly for η_2 agents when switching from exogenous to endogenous labor supply. In addition, the decline in rates of return is more severe in the exogenous labor supply scenario, and consequently welfare losses are stronger for all groups.

Table X: Welfare Consequences, Newborns in U.S.				
	Endog. Lab. Supply		Exog. Lab. Supply	
	η_1	η_2	η_1	η_2
k_1	-4.5%	-1.8%	-5.6%	-3.9%
k_2	-4.5%	-2.0%	-5.8%	-4.2%

7 Conclusions

In all major industrialized countries the population is aging, bringing with it a potentially large impact on the returns to the production factors capital and labor. This paper has documented that the welfare consequences from the declines in rates of return can be substantial, in the order of up to 5% in lifetime consumption for newborns in 2005. Allowing capital to freely flow between the OECD and the rest of the world may mitigate these costs substantially (but not eliminate them), if returns to capital in the rest of the world are initially higher than those in the OECD.

The welfare losses we document have to be traded off against the potential welfare gains from a longer (and healthier) life that is part of the source of the aging of the population in industrialized countries. While quantifying these welfare gains is beyond the scope of the current paper, our results should evidently not be interpreted as a statement that people living longer is a bad thing.

Another potentially beneficial side effect of a shrinking population (or a less rapidly growing population) that we have abstracted from may emanate from a reduction in the price of housing, assuming a supply that is at least somewhat inelastic. Since a serious quantitative evaluation of this effect requires an appropriate model of housing (choice) and thus the need of adding a continuous state variable to our model, carrying out such an analysis is beyond the scope of current computational feasibility, and thus left for future research.

References

[1] Abel, A. (2003), "The Effects of a Baby Boom on Stock Prices and Capital Accumulation in the Presence of Social Security," *Econometrica*, 71, 551-578.

[2] Altig, D., A. Auerbach, L. Kotlikoff, K. Smetters and J. Walliser (2001), "Simulating Fundamental Tax Reform in the U.S.," *American Economic Review*, 91, 574-595.

[3] Attanasio, O., S. Kitao and G. Violante (2006a), "Quantifying the Effects of the Demographic Transition in Developing Economies," *mimeo*, New York University.

[4] Attanasio, O., S. Kitao and G. Violante (2006b), "Global Demographic Trends and Social Security Reform," *mimeo*, New York University.

[5] Auerbach, A. and L. Kotlikoff (1987), *Dynamic Fiscal Policy*, Cambridge University Press, Cambridge.

[6] Bloom, D. and J. Williamson (1998), "Demographic Transitions and Economic Miracles in Emerging Asia," *World Bank Economic Review*, 12(3), 419-455.

[7] Börsch-Supan, A, A. Ludwig and J. Winter (2005), "Aging, Pension Reform, and Capital Flows: A Multi-Country Simulation Model," forthcoming, *Economica.*

[8] Brooks, R. (2003), "Population Aging and Global Capital Flows in a Parallel Universe," *IMF Staff Papers*, 50, 200-221.

[9] Carroll, C. (2005), "The Methods of Endogenous Gridpoints for Solving Dynamic Stochastic Optimization Problems," *CFS Working Paper* 2005/18.

[10] Conesa, J. and D. Krueger (1999), "Social Security Reform with Heterogeneous Agents," *Review of Economic Dynamics*, 2, 757-795.

[11] Conesa, J. and D. Krueger (2005), "On the Optimal Progressivity of the Income Tax Code," forthcoming, *Journal of Monetary Economics.*

[12] Deaton, A. (1991), "Saving and Liquidity Constraints," *Econometrica*, 59, 1221-1248.

[13] De Nardi, M., S. İmrohoroğlu and T. Sargent (1999), "Projected U.S. demographics and Social Security," *Review of Economic Dynamics*, 2, 575-615.

[14] Diamond, P. (1965), "National Debt in a Neoclassical Growth Model," *American Economic Review*, 55, 1126-1150.

[15] Domeij, D. and M. Floden (2005), "Population Aging and International Capital Flows," forthcoming, *International Economic Review.*

[16] Fehr, H., S. Jokisch and L. Kotlikoff (2005), "The Developed World's demographic Transition - The Role of Capital Flows, Immigration and

Policy," in R. Brooks and A. Razin (eds.) *Social Security Reform - Financial and Political Issues in International Perspective*, Cambridge University Press, Cambridge.

[17] Feroli, M. (2003), "Capital Flows Among the G-7 Nations: A Demographic Perspective," *mimeo*, Federal Reserve Board

[18] Hansen, G. (1993), "The Cyclical and Secular Behavior of the Labor Input: Comparing Efficiency Units and Hours Worked," *Journal of Applied Econometrics*, 8, 71-80.

[19] Hendriksen, E. (2002), "A Demographic Explanation of U.S. and Japanese Current Account Behavior," *mimeo*, Carnegie Mellon University.

[20] Huang, H., S. İmrohoroğlu and T. Sargent (1997), "Two Computations to Fund Social Security," *Macroeconomic Dynamics*, 1, 7-44.

[21] İmrohoroğlu, A., S. İmrohoroğlu and D. Joines (1995), "A Life Cycle Analysis of Social Security," *Economic Theory*, 6, 83-114.

[22] İmrohoroğlu, A., S. İmrohoroğlu and D. Joines (1995), "Social Security in an Overlapping Generations Economy with Land," *Review of Economic Dynamics*, 2, 638-665.

[23] Ludwig, A. (2005a), "The Gauss-Seidel-Quasi-Newton Method: A Hybrid Algorithm for Solving Dynamic Economic Models," forthcoming, *Journal of Economic Dynamics and Control.*

[24] Ludwig, A. (2005b), "Moment Estimation in Auerbach-Kotlikoff Models: How Well Do They Match the Data?," *mimeo*, MEA, University of Mannheim.

[25] Obstfeld, M. and K. Rogoff (1995), "The Intertemporal Approach to the Current Account," in G. Grossman and K. Rogoff (eds.) *Handbook of International Economics*, Vol. 3, Elsevier, Amsterdam.

[26] Storesletten, K., C. Telmer and A. Yaron (2004), "Consumption and Risk Sharing over the Life Cycle," *Journal of Monetary Economics,* 51, 609-633.

[27] United Nations (2002), "World Population Prospects: The 2002 Revision," United Nations Population Division, United Nations, New York.

A Details of the Demographic Projections

For each country $i \in \{1, \ldots, I\}$ we base our demographic data on the official demographic data and projections by the United Nations (United Nations, 2002). Starting from a given initial age-distribution of population, $N_{0,\bar{j},i}$, in year 1950 for actual age $\bar{j} \in \{0, \ldots, 96\}$ demography in each year t is given recursively by

$$
\begin{aligned}
N_{t+1,\bar{j}+1,i} &= N_{t,\bar{j},i}(s_{t,\bar{j},i} + m_{t,\bar{j},i}), \quad m_{t,\bar{j},i} = 0 \text{ for } \bar{j} > 19 \\
N_{t+1,0,i} &= \sum_{\bar{j}=15}^{50} f_{t,\bar{j},i} N_{t,\bar{j},i}
\end{aligned}
$$

where $m_{t,\bar{j},i}(f_{t,\bar{j},i})$ denotes time, age and country specific migration (fertility) rates. Our assumption, that migration rates are zero for ages above 19 allows us to treat newborns and immigrants in the economic model alike, compare Section 3.1.

The United Nations provide demographic data on $N_{t,\bar{j},i}$, $s_{t,\bar{j},i}$ and $f_{t,\bar{j},i}$ on an annual basis for the years 1950-2050, but for age-groups of five only. We interpolate the initial distribution of the population, $N_{1950,\bar{j},i}$, and the data on $s_{t,\bar{j},i}$ and $f_{t,\bar{j},i}$ for all $t \in \{1950, \ldots, 2050\}$ between age-groups to get age-specific data. As for migration we use the UN data on aggregate migration, $M_{t,i}$, and assume that migration numbers are equally distributed across ages for $\bar{j} \in \{0, \ldots, 19\}$. These approximations result in a decent fit of our demographic model to the official UN figures.

We further forecast demography beyond the UN forecasting horizon until 2300. First, while holding fertility rates constant, we assume that life-expectancy continues to increase at constant rates until year 2100. We then hold age-specific survival rates constant and assume that fertility rates adjust such that the number of newborns is constant in each successive year until 2200. This adjustment procedure implies that stationary population numbers are reached in year 2200. To support the steady state in our economic model, we hold demography constant for an additional 100 years until 2300.

B Computational Details

B.1 Household Problem

The idea is to iterate on the Euler equation, using ideas developed in Carroll (2005). The dynamic programming problem of the household reads as

$$\begin{aligned}
& W(t,j,k,i,\eta,a) \qquad (12)\\
&= \max_{c,a',1-l}\{u(c,1-l)+\beta s_{t,j,i}\sum_{\eta'}\pi(\eta'|\eta)W(t+1,j+1,k,i,\eta',a')\}\\
&\text{s.t. } c+a' = \begin{cases}(1-\tau_{t,i})w_{t,i}\theta_k\varepsilon_j\eta l+(1+r_t)(a+h_{t,i}) & \text{for } j<jr\\ b_{t,j,k,i}+(1+r_t)(a+h_{t,i}) & \text{for } j\geq jr\end{cases}\\
& a',c\geq 0 \text{ and } l\in[0,1]
\end{aligned}$$

where t indexes time, j indexes age, k indexes type, i indexes country, η denotes the idiosyncratic income shock and a denotes asset holdings.

Following Deaton (1991), we denote by x "cash-on-hand" the maximum amount of resources available,

$$x = \begin{cases}(1-\tau_{t,i})w_{t,i}\theta_k\varepsilon_j\eta+(1+r_t)(a+h_{t,i}) & \text{for } j<jr\\ b_{t,j,k,i}+(1+r_t)(a+h_{t,i}) & \text{for } j\geq jr\end{cases}$$

Now, rewrite the Bellman equation as

$$\begin{aligned}
& V(t,j,k,,i\eta,x)\\
= & \max_{c,1-l}\{u(c,1-l)+\beta s_{t,j,i}\cdot\\
& \sum_{\eta'}\pi(\eta'|\eta)V(t+1,j+1,k,i,\eta',(1+r_{t+1})\left(x-c-\theta_k\varepsilon_j\eta w_{t,i}(1-\tau_{t,i})(1-l)\right)+y')\},
\end{aligned}$$

where

$$y' = \begin{cases}w_{t+1,i}(1-\tau_{t+1,i})\theta_k\varepsilon_{j+1}\eta'+(1+r_{t+1})h_{t+1,i} & \text{for } j<jr\\ b_{t+1,j+1,k,i}+(1+r_{t+1})h_{t+1,i} & \text{for } j\geq jr\end{cases}$$

The resulting inter-temporal Euler equation for consumption reads as

$$\begin{aligned}
u_c & \geq \beta s_{t,j,i}(1+r_{t+1})\sum_{\eta'}\pi(\eta'|\eta)V_{x'}(t+1,j+1,k,i,\eta',x')\\
& = \text{ if } a'>0 \qquad (13)
\end{aligned}$$

and the intratemporal Euler equation for leisure as

$$\begin{aligned}
u_{1-l} & \geq \beta s_{t,j,i}(1+r_{t+1})\sum_{\eta'}\pi(\eta'|\eta)V_{x'}(t+1,j+1,k,i,\eta',x')w_{t,i}(1-\tau_{t,i})\theta_k\varepsilon_j\eta\\
& = \text{ if } l>0 \qquad (14)
\end{aligned}$$

and the envelope condition reads as

$$V_x(t, j, k, i, \eta, x) = u'(c) \tag{15}$$

We can therefore define the intratemporal Euler equation between consumption and leisure as

$$\begin{aligned} u_{1-l} &\geq u_c w_{t,i}(1-\tau_{t,i})\theta_k \varepsilon_j \eta \\ &= \text{ if } l > 0 \text{ and } a' > 0 \end{aligned} \tag{16}$$

from which, for the family of Cobb-Douglas utility functions, we can get an explicit solution for leisure in terms of consumption and wages as

$$\begin{aligned} 1-l &= 1-l\left(c, w_{t,i}(1-\tau_{t,i})\theta_k \varepsilon_j \eta\right) \\ \text{if } & l > 0 \text{ and } a' > 0 \end{aligned} \tag{17}$$

Our algorithm operates on (13),(15) and (17).

1. Define a type-independent grid for savings

$$\mathcal{G}^A = \{0, a_2, \ldots, a_{na}\}$$

2. Define a type-dependent grid on x for the last generation

$$\mathcal{G}^S_{J,k} = \{x_{1,k}, \ldots, x_{na,k}\}$$

   Choose $x_{nx} > a_{na}$ and $x_1 = x_{\min} > 0$, but small. Furthermore let $nx = na + 1$.

3. From economic theory it follows that

$$\begin{aligned} c(t, J, k, i, \eta, x) &= x \\ l(t, J, k, i, \eta, x) &= 0 \\ a'(t, J, k, i, \eta, x) &= 0 \end{aligned}$$

   for all $x \in \mathcal{G}^S_{J,k}$. From (15)

$$\begin{aligned} V'_{x'}(t, J, k, i, \eta, x) &= u'_c(t, J, k, i, \eta, x) \\ V_x(t, J, k, i, \eta, x) &= u_c(t, J, k, i, \eta, x) \end{aligned}$$

4. Now iterate on j, $j = nj-1, \ldots, 1$. Given that the function $V'_{x'}(t+1, j+1, k, i, \eta, x)$ is known from the previous step, do the following

   (a) For all $a' \in \mathcal{G}^A$, solve equation (13) for c using equation (17) for numbers $(c_1, \ldots, c_{na})$. Since

$$x' \notin \mathcal{G}^S_{j+1,k}$$

   in general, this involves interpolation of the function V. For reasons of accuracy, we follow the suggestions of Carroll (2005) and interpolate on an appropriate transform of V' which is approximately linear.

(b) Equipped with the consumption numbers, calculate leisure by solving equation (17) for number $(l_1, \ldots, l_{na})$.. If $l < 0$ then go back to (a) and calculate consumption for $l = 0$.

(c) Equipped with the consumption and leisure numbers, define the grid $\mathcal{G}^S_{j,k}$ by

$$
\begin{aligned}
x_1 &= x_{\min} \\
x_{p+1} &= \begin{cases} a_p + c_p + (1 - l_p) w_{t,i} (1 - \tau_{t,i}) \theta_k \varepsilon_j \eta & \text{for } j < jr \\ a_p + c_p + b_{t,j,k,i} & \text{for } j < jr \end{cases} \\
&\quad \text{for } p = 1, \ldots, na
\end{aligned}
$$

and the consumption functions

$$
\begin{aligned}
c(t, j, k, i, \eta, x_1) &= x_{\min} \\
c(t, j, k, i, \eta, x_p) &= c_p \text{ for } p = 2, \ldots, nx \\
l(t, j, k, i, \eta, x_p) &= l_p \text{ for } p = 1, \ldots, nx \\
a(t, j, k, i, \eta, x_1) &= 0 \\
a'(t, j, k, i, \eta, x_{p+1}) &= a_p \text{ for } p = 1, \ldots, na
\end{aligned}
$$

(d) Update for all $x \in \mathcal{G}^S_{j,k}$

$$
\begin{aligned}
& V_x(t, j, k, i, \eta, x) \\
= \ & u_c(c(t, j, k, i, \eta, x), 1 - l(t, j, k, i, \eta, x)) \\
& V(t, j, k, i, \eta, x) \\
= \ & u(c(t, j, k, i, \eta, x), 1 - l(t, j, k, i, \eta, x)) + \beta s_{t,j,i} \\
& \sum_{\eta'} \pi(\eta' | \eta) V(t + 1, j + 1, k, i, \eta', x')
\end{aligned}
$$

The updating of the value function again involves interpolation.

B.2 The Aggregate Model

For a given $r \times 1$ vector $\vec{\Psi}$ of structural model parameters, we first solve for an "artificial" initial steady state in period $t = 0$ which gives an initial distribution of assets. We thereby presume that households assume prices to remain constant for all periods $t \in \{0, \ldots, T\}$ and are then surprised by the actual price changes induced by the transitional dynamics. Next, we solve for the final steady state of our model which is reached in period T and supported by our demographic projections, see Appendix A. For both steady states, we solve for the equilibrium of the aggregate model by iterating on the $m \times 1$ steady state price vector $\vec{P}_{ss} = [p_1, \ldots, p_m]'$. p_1 is the world marginal product of capital, $r_t + \delta$, $p_2, \ldots, p_{I+1}$ are social security contribution (or replacement) rates of each country, $\tau_{t,i}(\rho_{t,i})$, and $p_{I+2}, \ldots, p_{2I+1}$ are accidental bequests (as a ratio of wages) in each country,

$\frac{h_{t,i}}{w_{t,i}}$. Notice that all elements of $\vec{P}_{ss}$ are defined such that they are constant in the steady states.

Solution for the steady states of the model involves the following steps:

1. In iteration q for guess $\vec{P}^q_{ss}$ solve the household problem.

2. Next aggregate across all households in all countries to get the world-wide aggregate capital stock, $\sum_i K_{t,i}$. Define by $\kappa_{t,i}$ the capital stock per efficiency unit in country i,

$$\kappa_{t,i} = \frac{K_{t,i}}{Z_i^{1/(1-\alpha)} A_t L_{t,i}} = \kappa_t \ \forall i,$$

which is constant across all countries by our assumption of perfect capital mobility and the market market clearing condition for the world interest rate, $r_t + \delta = \alpha \kappa_t^{\alpha-1}$. It follows that

$$\sum_i K_{t,i} = \kappa_t \sum_i Z_i^{1/(1-\alpha)} A_t L_{t,i}.$$

We use this condition to form an update of the world marginal product of capital, $\tilde{p}_1$.

3. Aggregate across all households to get aggregate labor supply in each country, $L_{t,i}$. Update social security contributions (or replacement rates) using the social security budget constraint, equation (9). Calculate bequests in each country by equation (11).

4. Collect the updated variables in $\tilde{\vec{P}}_{ss}$ and notice that $\tilde{\vec{P}}_{ss} = H(\vec{P}_{ss})$, where H is a vector-valued non-linear function.

5. Define the root-finding problem $G(\vec{P}_{ss}) = \vec{P}_{ss} - H(\vec{P}_{ss})$, where G is a vector-valued non-linear function, and iterate on $\vec{P}_{ss}$ until convergence. We use Broyden's method to solve the problem and denote the final approximate Jacobi matrix by B_{ss}.

Next, we solve for the transitional dynamics by the following steps:

1. Use the steady state solutions to form a linear interpolation to get the starting values for the $m(T-2) \times 1$ vector of equilibrium prices, $\vec{P} = [\vec{p}_1', \ldots, \vec{p}_m']'$, where $p_i, i = 1, \ldots, m$ are vectors of length $(T-2) \times 1$.

2. In iteration q for guess $\vec{P}^q$ solve the household problem. We do so by iterating backwards in time for $t = T-1, \ldots, 2$ to get the decision rules and forward for $t = 2, \ldots, T-1$ for aggregation.

3. Update variables as in the steady state solutions and denote by $\tilde{\vec{P}} = H(\vec{P})$ the $m(T-2) \times 1$ vector of updated variables.

4. Define the root-finding problem as $G(\vec{P}) = \vec{P} - H(\vec{P})$. Since T is large, this problem is substantially larger and we use the Gauss-Seidel-Quasi-Newton algorithm suggested in Ludwig (2005a) to form and update guesses of an approximate Jacobi matrix of the system of $m(T-2)$ non-linear equations.

B.3 Calibration of Structural Model Parameters

Calibration of structural model parameters is based on a procedure suggested in Ludwig (2005b). We split the $r \times 1$ vector of structural model parameters, $\vec{\Psi}$, as $\vec{\Psi} = \left[(\vec{\Psi}^e)', (\vec{\Psi}^f)'\right]'$. $\vec{\Psi}^f$ is a vector of predetermined (fixed) parameters, whereas the $e \times 1$ vector $\vec{\Psi}^e$ is estimated by minimum distance (unconditional matching of moments using e moment conditions). Denote by

$$u_t(\vec{\Psi}^e) = y_t - f(\vec{\Psi}^e) \text{ for } t = 0, \ldots, T_0$$

the GMM error as the distance between data, y_t, and model simulated (predicted) values, $f(\vec{\Psi}^e)$.

Under the assumption that the model is correctly specified, the restrictions on the GMM error can be written as

$$\mathbb{E}[u_t(\vec{\Psi}^e_0)] = 0,$$

where $\tilde{\Psi}^e_0$ denotes the vector of true values. Denote the sample averages of u_t as

$$g_{T_0}(\vec{\Psi}^e) \equiv \frac{1}{T_0 + 1} \sum_{t=0}^{T_0} u_t(\vec{\Psi}^e). \tag{18}$$

We estimate the elements of $\vec{\Psi}^e$ by setting these sample averages to zero (up to some tolerance level).

In our economic model, the vector $\vec{\Psi}^e$ is given by

$$\vec{\Psi}^e = [g, \delta, \alpha, Z_i, \ldots, Z_I, \beta, \omega_i, \ldots, \omega_I]'.$$

We estimate the structural model parameters using data from various sources for the period 1950, ..., 2004, hence $T_0 = 54$. To identify the structural model parameters of the production function, g, δ, α, we use NIPA data for the U.S. on GDP, fixed assets, depreciation, wages and labor supply. Since our economic model restricts the capital-output ratio to be equal across countries, we restric $Z_1 = 1$ and can estimate Z_i for $i > 1$ using data on labor productivity of country i relative to the U.S.. The remaining structural model parameters, $\beta, \omega_i, \ldots, \omega_I$, are estimated by simulation. We estimate β by setting to zero the average distance between the simulated and the actual capital-output ratio and $\omega_i, \ldots, \omega_I$ by setting to zero the average distance between simulated and actual average hours worked.

The predetermined parameter of our model is the coefficient of relative risk aversion, σ, which we set to one. In addition, we impose the restriction that the parameters, $\alpha, \delta, g, \beta, \sigma$, are constant across countries.