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Estimating Potential Output in the Republic of Croatia Using a Multivariate Filter

Nikola Bokan and Rafael Ravnik

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Abstract

This paper estimates potential output in the Republic of Croatia for the period between the first quarter of 2000 and the fourth quarter of 2010, using a combination of a multivariate Kalman filter and the regularised maximum likelihood method. For the estimation of potential output a dynamic macroeconomic model was developed, similar to that used in Benes et al. (2010), in which core inflation is the key determinant of potential output and the output gap. This is why potential output, as defined herein, can be construed as the level of output that can be sustained in the long run without creating either upward or downward pressures on core inflation. Apart from the aforementioned core inflation, the model includes some other relevant economic series, such as the unemployment rate, retail trade, industrial production index and current account deficit, which, if ignored, as in the case of univariate filters, can result in a potential output estimation bias.

The estimation results show that output was below its potential level until the second quarter of 2002, after which it remained above the potential level for almost seven years. In the second quarter of 2009, however, output sank below its potential level, where it remained until the end of the reference period. During the last observed period, both actual and potential output levels declined.

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Keywords:

Croatia, potential output, output gap, multivariate filter

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1 Introduction

For a large number of years, potential output estimation has been one of the major challenges to both economic analysis and economic policy implementation. The problem consists in an inability to measure (observe) directly the value of potential output, which then obstructs the calculation of aggregate demand surplus (deficit), i.e. the output gap. The importance of the potential output estimate is reflected in the fact that the mechanism of monetary policy influence on inflation largely depends on the adjustment of monetary policy to movements in aggregate demand surplus, as well as on the successful achievement of the set goals. Hence, the measuring and understanding of the potential output values are key determinants of successful monetary policy implementation. Since potential output movements are important indicators of business cycles, an adequate method for their calculation is also important for a successful fiscal policy implementation. Another reason for estimating potential output in the Republic of Croatia is the absence of any recent domestic research into this matter. Only Vrbanc (2006) estimated potential output, applying the production function method, while no other relevant methods were applied.

The definition of potential output and attempts to calculate it date back to the research by Okun (1962). During the past forty years, a great number of different methods were developed to improve its estimation. One of the most frequently used was that postulating a connection between the available factors of production such as labour, capital, resources and technology and output, through a certain production function, where the current level of potential output is defined as a level determined by the current value of fixed production inputs and a "sustainable" value of variable inputs. As indicated in Benes and N'Diaye (2004), although the production function involves a connection between the labour and goods markets, in practice, the use of the production function does not significantly improve the potential output estimate, because uncertainty of the potential output estimate turns into uncertainty of the total factor productivity estimate.

More recent approaches to potential output calculation are based on some of the statistical filtering methods, i.e. the use of specific methods of time series analysis, resulting in an estimated time series representing a trend. This time series is then interpreted as a presentation of equilibrium values of the original time series. In its simplest form, filtering can only be applied to one time series (univariate filter), where trend lines are only calculated on the basis of information from the analysed series.

One of the most-used univariate filters is the Hodrick-Prescott (HP) filter¹, due primarily to the simplicity of its application. In this filter, trends represent two-sided moving averages, which are calculated by using both previous and future data in each period. When using the HP filter and similar univariate filters, the value of the parameter determining the smoothness of the series must be assumed in advance. This enables the researcher to control the end result by arbitrarily selecting values for the said parameter, while the methodology itself cannot respond to the question of how smooth the series should be, on the basis of a built-in objective

1 For more about univariate filters, see Hodrick and Prescott (1981), or Baxter and King (1999).

criterion. Given the lack of a formal criterion within the methodology, the possibility remains of determining the parameter value on the basis of the researcher's assumptions or by defining the estimation problem in broader terms, where the parameter value will be determined on the basis of a criterion outside the original methodology. For example, in the case of potential output, Laxton and Tetlow (1992), and Benes and N'Diaye (2004) argue that, when the HP filter is used, the smoothness of the series should be determined according to the nature of shocks in the observed economy. That is, if shocks on the aggregate demand side are more pronounced, potential output will not follow the actual output movements, so that high values of the parameter determining the smoothness of the series should be used. Conversely, should shocks on the aggregate supply side prevail, the potential output series will follow the actual output series, so that it would be appropriate to use lower values of the smoothness parameter.

However, the use of univariate filters involves a lot of problems. One of the drawbacks of univariate filters is the inaccuracy of estimated series, which increases towards the end of the sample period. The inaccuracy is shown when revising the series due to a change in the sample size, which is the consequence of the structure of such filters. More specifically, given the unavailability of future observed values at the end of the sample period, the HP filter forecasts them, so that if the sample is enlarged by new data, these estimates necessarily have to be revised.

Another problem arises from the fact that univariate filters ignore some important economic information included in other time series, and hence their use can result in a strong estimation bias. As an example of this, Benes et al. (2010) mention a situation observed in developed countries, where in the last ten years the monetary policy was primarily focused on fighting inflation, which led to a prolonged negative potential output gap. If this information is ignored, the potential output and, consequently, the output gap will be underestimated.

It is due to these univariate filter-related problems that the multivariate filter methodology is used in this research, and the average revision of the filtered series for the HP filter is compared with the revision obtained using the multivariate filter, in order to draw conclusions about the stability of the obtained potential output series for both methods. The main purpose of a multivariate filter is to use information from not only the data on an economy's output, but also the data on some other economically relevant time series. It should be noted that, in this research, estimated trend values are interpreted as equilibrium values, closely connected with inflation. More precisely, the trend values used under this methodology for the calculation of the values of gaps within an economy, which in turn represent the main determinants of inflation dynamics and other potentially significant economic categories, are defined in a manner that enables an at least partial description of movements in these categories. Hence, it would be incorrect to interpret the obtained potential output values as maximum values that a certain economy could reach through the optimal and unrestricted employment of available resources.

For this reason, potential output is defined in this paper as "a level of output that may be sustained indefinitely, without creating a tendency for inflation to rise or fall" (Benes et al., 2010, p. 5), where it is expected that inflation will be stable in a period when actual output is about equal to potential output, whereas inflation increases (decreases) would relate to periods of positive (negative) output gaps. However, there is no reason why the analysis should be limited to additional information incorporated in the observed inflation time series. This is exactly why in this research, the multivariate filter involves interactions between actual output and potential output, as well as inflation, unemployment, industrial production and current account balance, in the form of a small dynamic macroeconomic model, where unknown model parameters and unobserved time series (potential output, potential industrial production and unemployment) are simultaneously estimated. Benes et al. (2010, p. 6) note that this approach "has a flexibility, which allows the estimated growth of potential to vary with an array of recent information, while at the same time taking into account the more stable trends evident in long-run time series."

The current economic situation in the Republic of Croatia underlines the importance of an accurate estimate and analysis of potential output and of the influence of the financial crisis on its movements and gap. In the period between 2003 and 2008, the Croatian economy recorded relatively high growth rates in an environment of stable inflation. However, the situation changed radically, due to the global financial crisis. A fall in exports and a halt in capital inflows led to a decline in consumption and investment and, consequently, in overall output. While part of this downturn can be attributed to cyclical factors, the other part was certainly due to slower growth and even a fall in the level of potential GDP, which can be the result of reduced capital inflows and higher prices of capital. This was partly the consequence of a change in the global risk aversion, but also of an increase in government borrowing in foreign markets. Moreover, the crisis has most probably led to a fall in the level of capital, due to the failure of some enterprises and suspension of investment projects, notably in the non-tradable goods sectors.

This effect of the financial crisis on the long-term trend in Croatia's output is clearly shown in the obtained results, which suggest a decline not only in the cyclical component but also in potential output during 2009 and 2010. Furthermore, the results clearly show the interaction between the output gap and inflation rate, while the actual maximum (and minimum) output gaps also correspond with the actual maximum (and minimum) inflation rates. The advantage of using the multivariate filter approach to potential output estimation over that based on a univariate filter is confirmed by a test of robustness to any change in the sample.

The paper is organised as follows: The second chapter describes the structure of a small macroeconomic model used for the calculation of potential output in the Republic of Croatia. The third chapter deals with the methodology applied in estimating model parameters, as well as the data set used in the analysis. The fourth chapter analyses the features of the estimated potential output, output gap, interaction between inflation and output gap, the effect of the output gap change on actual output, and examines the relative robustness of the output gap. The last chapter is the conclusion.

2 Model description

Following Benes et al. (2010) and El-Ganainy and Weber (2010), we define three different gaps observed in the model: the output gap, unemployment gap and industrial production gap. We will expand the model by specifying retail trade and current account deficit gaps. The output gap is defined as the difference between the logarithms of actual (realised) output, Y_i , and potential output, $\overline{Y_i}$:

$$y_t = 100 \cdot \left[\log(Y_t) - \log(\overline{Y_t}) \right] \tag{1}$$

The unemployment gap is defined as the difference between the equilibrium unemployment rate (NAIRU), U_t , and the actual unemployment rate, U_i :

$$u_t = \overline{U}_t - U_t \tag{2}$$

The gaps for the industrial production and retail trade indices are defined using the annual growth rate². The gap in the annual industrial production growth rate, G_{i}^{indpr} , is defined as the difference between the annual growth rate of the actual industrial production, G_{i}^{NDPR} , and the equilibrium growth rate, $G_{i}^{\overline{NDPR}}$.

$$G_t^{indpr} = G_t^{INDPR} - G_t^{\overline{INDPR}}$$
(3)

As in the case of industrial production, we will define the gap in the annual growth rate of retail trade as the difference between the actual annual growth rate and the corresponding equilibrium annual growth rate, as shown by the following equation:

² Gaps are expressed as annual growth rates only for the two specified variables, whereas for other variables they are expressed as levels. Retail trade and the industrial production index are used in the specified form in order to increase the information set available in the Kalman filter and thus improve the reliability of the gap and potential output estimates.

$$G_t^{tnm} = G_t^{TNM} - G_t^{\overline{TNM}} \tag{4}$$

The current account deficit gap is defined in the similar way, but it is expressed in levels rather than growth rates:

$$dtr_t = DTR_t - DTR_t \tag{5}$$

The annual core inflation rate, $\pi 4_t$, defined as the rate of change in the core consumer price index relative to the same quarter of the previous year, is depicted by the following equation:

$$\pi 4_{t} = \pi 4_{t-1} + \beta y_{t} + \Omega (y_{t} - y_{t-1}) + \varepsilon_{t}^{\pi 4}$$
(6)

In addition to the output gap, representing a typical short-run trade-off, and lagged inflation, the equation includes the first difference of output gap, which introduces a certain degree of rigidity into the economy adjustment process. As explained in Benes et al. (2010), with y_t negative and $y_t - y_{t-1}$ positive, i.e. in the case of coming out of a recession, such a change in the gap would lessen the effect of the gap itself on the inflation rate, and would thus capture the effects of the adjustment speed limitation, due to capacity constraints in some sectors of the economy. It should be noted that, with the lagged inflation, the coefficient equals one. The lagged inflation rate can thus be interpreted as a proxy for inflation expectations. Furthermore, it should be borne in mind that this restriction implies a complete absence of any long-run trade-off between output and inflation.

In order to link the output gap to the unemployment gap, we use a modified version of Okun's law, as in the following equation:

$$u_t = \phi_1 u_{t-1} + \phi_2 y_t + \varepsilon_t^u \tag{7}$$

The modification is due to an empirical fact suggesting a lag between a change in output and the resulting change in unemployment.

Benes et al. (2010) define a relationship between the output gap and capacity utilisation similar to that in Okun's law, implicitly assuming that manufacturing capacity utilization contains information important for the estimation of potential output and the output gap. Given the lack of capacity utilisation time series in the Republic of Croatia, we will use the industrial production and retail trade indices, which can be reasonably assumed to contain important information necessary for reliable potential output and output gap estimates. The links between the gap in the annual industrial production growth and output gap and between the gap in the annual growth rate of retail trade and output gap are depicted by the following equations:

$$G_{t}^{indpr} = \kappa_1 G_{t-1}^{indpr} + \kappa_2 y_t + \varepsilon_t^{indpr}$$

$$\tag{8}$$

$$G_t^{tnm} = \kappa_3 G_{t-1}^{tnm} + \kappa_4 y_t + \varepsilon_t^{tnm}$$
⁽⁹⁾

We will also define the link between the current account deficit gap and output gap, using the following equation:

$$dtr_t = \kappa_5 dtr_{t-1} + \kappa_6 y_t + \varepsilon_t^{dr} \tag{10}$$

It can be expected that κ_2 , κ_4 and κ_6 will be greater than one, according to the assumption that fluctuations in industrial production, retail trade and current account deficit will be more pronounced than the fluctuation in aggregate output.

After defining the model structure with respect to various gaps, we continue by specifying equations that depict the movements in equilibrium values. The equilibrium unemployment rate (NAIRU), depends not only

4

on its value in the previous period, \overline{U}_{t-1} , but also on the deviation of this value from the steady-state value of unemployment, $\overline{U}_{t-1} - U^{ss}$, and on the previous-period's output gap. The specification additionally includes two stochastic elements, one relating to a transitory shock, $\mathbf{\varepsilon}_{t}^{\overline{U}}$, and the other to a persistent shock on the equilibrium unemployment rate, $G_{t}^{\overline{U}}$.

$$\overline{U}_{t} = \overline{U}_{t-1} + G_{t}^{\overline{U}} - \frac{\omega}{100}_{y_{t-1}} - \frac{\lambda}{100} (\overline{U}_{t-1} - U^{SS}) + \varepsilon_{t}^{\overline{U}}$$
(11)

We assume that the stochastic element, $G_t^{\overline{U}}$ follows an autoregressive process of order one, defined as:

$$G_t^{\overline{U}} = (1 - \alpha) G_{t-1}^{\overline{U}} + \varepsilon_t^{G^{\overline{U}}}$$
(12)

While such a specification allows for a persistent influence of the shock on the NAIRU, it assumes a fixed steady-state level of unemployment in the long run.

Potential output is depicted by the following equation:

$$\overline{Y}_{t} = \overline{Y}_{t-1} - \theta(\overline{U}_{t} - \overline{U}_{t-1}) - (1 - \theta)(\overline{U}_{t-1} - \overline{U}_{t-20})/19 + G_{t}^{\overline{Y}}/4 + \varepsilon_{t}^{\overline{Y}}$$
(13)

The equation is specified by potential output as a function of the potential output from the previous period, the trend rate of potential output growth, $G_t^{\overline{Y}}$, shock, $\varepsilon_t^{\overline{Y}}$, and short-term and medium-term changes in the equilibrium unemployment rate. The change in the equilibrium unemployment rate, $\overline{U}_t - \overline{U}_{t-1}$, represents the impact on the equilibrium output growth via a Cobb-Douglas production function, where θ , is the share of labour in total production. The second element, $\overline{U}_{t-1} - \overline{U}_{t-20}$, the 19-period difference in unemployment rate, \overline{U}_t , will result in a θ decline in potential output in the same period. However, the negative trend will continue for the following 19 periods, finally leading to a long-run decline in potential output of 1%.

The trend rate of potential output growth is depicted by the following stochastic process:

$$G_{t}^{\overline{Y}} = \tau G_{SS}^{\overline{Y}} + (1 - \tau) G_{t-1}^{\overline{Y}} + \varepsilon_{t}^{G^{\overline{Y}}}$$
(14)

The equation implies that the growth rate is not constant, but is allowed to deviate from the steady-state growth rate, these deviations being serially correlated.

As in the preceding equation, the equilibrium values of the annual growth rate of industrial production, annual growth rate of retail trade and of the current account deficit are summarised in the following stochastic processes:

$$G_{t}^{\overline{INDPR}} = (1-a)G_{t-1}^{\overline{INDPR}} + aG^{\overline{INDPR}} - \varepsilon_{t}^{\overline{INDPR}}$$
(15)

$$G_{t}^{\overline{\text{TNM}}} = (1-c)G_{t}^{\overline{\text{TNM}}} + cG_{t}^{\overline{\text{TNM}},SS} + \varepsilon_{t}^{\overline{\text{TNM}}}$$
(16)

$$\overline{DTR}_{t} = (1-d)\overline{DTR}_{t-1} + dG^{DTR_{SS}} + \varepsilon_{t}^{\overline{DTR}}$$
(17)

The expected inflation target, $\pi 4_{t}^{LTE}$, is also included in the model:

$$\pi 4_{t}^{LTE} = \pi 4_{t-1}^{LTE} + \varepsilon_{t}^{\pi 4^{LTE}} \tag{18}$$

In the event of a change in the monetary policy regime or during volatile regimes, variance in $\varepsilon_i^{\pi 4^{LTE}}$ would be higher than in a stable regime. In the event of a fixed exchange rate, where no stable inflation target can be set, variance would also be high.

Conventional inflation-targeting models include a specification of the interest rate reaction function, as an instrument for keeping inflation on target, with the Phillips curve being a key element of a sophisticated mechanism through which the interest rate influences core inflation. In effect, monetary policy influences core inflation by influencing the output gap. In our model, the said effect is slightly modified and is depicted by the following output gap equation, where a positive inflation deviation from the target in the previous period negatively affects the current output gap:

$$y_{t} = \rho_{1} y_{t-1} - \frac{\rho_{2}}{100} (\pi 4_{t-1} - \pi 4_{t-1}^{LTE}) + \varepsilon_{t}^{y}$$
(19)

It should be noted that this specification is consistent with a broad range of monetary regimes. For example, in a fixed nominal exchange rate regime, increased demand in the previous period results in a higher inflation rate and, consequently, an appreciation of the real exchange rate, which produces the said negative effect on output gap in the current period.

3 Estimation methodology and data used

As in Benes et al. (2010), the model is estimated using a combination of a multivariate Kalman filter and the regularised maximum likelihood method, which is a special case of the Bayesian estimation method.^{3,4} In applying the regularised maximum likelihood method, we use the observed data and prior distributions and parameters to obtain final posterior distributions and parameters.⁵ This method is used because it allows the introduction of the said assumptions, thus preventing the parameters from wandering into nonsensical regions. Such a priori limitations are particularly useful in such systems where there is a possibility of insufficient data for the estimation of individual parameters.

Generally, regularisation in estimating the parameters of a system means the modification of criteria measuring the fit of a model, by adding an expression containing prior assumptions. There is a series of criteria used in the literature, the most frequent of which are the criteria defining prediction error or likelihood. The mathematical formulation of prior assumptions is multiplied by a penalising term so that it can be used for controlling the intensity of the influence of prior assumptions on the final result. The final result of regularisation is an estimated value representing a compromise between the value minimising the error or maximising likelihood and that which is close to the prior. In formal terms, the criterion function in the event of applying the criterion of prediction error minimisation can be the following:

$$V_{N}(\boldsymbol{\theta}; Y^{N}) = \frac{1}{N} \sum_{t=1}^{N} l(\boldsymbol{\varepsilon}_{F}(t, \boldsymbol{\theta}))$$

where θ is a vector of unknown parameters, *Y* represents used data, *N* is the sample size and $\varepsilon_F(t, \theta)$ is a series of prediction errors, filtered by means of a stable linear filter defined as:

$$\varepsilon_F(t,\theta) = L(q)\varepsilon(t,\theta)$$

A prediction error is defined as:

$$\varepsilon(t, \theta) = y(t) - \hat{y}(t \mid \theta)$$

where \hat{y} can be defined as the function of past data and unknown parameters:

³ The estimation was made using the IRIS toolbox in MATLAB.

⁴ For more about the Kalman filter, see Harvey (1989) and Hamilton (1994).

⁵ For more about this methodology, see Ljung (1999) and Ljung et al. (1992).

$$\hat{y}(t | \boldsymbol{\theta}) = g(t, Y^{t-1}; \boldsymbol{\theta})$$

The objective is to minimise the said criterion function with respect to θ in order to estimate the vector of unknown parameters:

$$\hat{\boldsymbol{\theta}}_{\scriptscriptstyle N} = \arg\min_{\boldsymbol{\theta}} V_{\scriptscriptstyle N}(\boldsymbol{\theta}; Y^{\scriptscriptstyle N})$$

If regularisation is applied, the criterion function can be extended as follows:

$$W_N^p(\mathbf{\Theta}; Y^N) = V_N(\mathbf{\Theta}; Y^N) + p \times l(\mathbf{\Theta})$$

where p is the aforementioned penalisation, and l a function that can be defined, in its special form, as the square of absolute difference between an unknown parameter and the prior:

$$l(\mathbf{\theta}) = |\mathbf{\theta} - \overline{\mathbf{\theta}}|^2$$

Parameters are estimated in the same way as in a case with no penalisation, except that the function to be minimised is now the extended function $W_N^p(\theta; Y^N)$:

$$\hat{\boldsymbol{\Theta}}_{N}^{p} = \arg\min_{\boldsymbol{\Theta}} W_{N}^{p}(\boldsymbol{\Theta}; Y^{N})$$

The last three equations clearly show how *p* can be used to control the power of $l(\theta)$, so that, for example, in the extreme case where p = 0, $W_N^p(\theta; Y^N) = V_N(\theta; Y^N)$, i.e. the result is equal to that obtained without applying regularisation.

Ljung (1999) presents two main advantages of system regularisation. The first is that it facilitates the minimisation of the criterion function in a case where it is difficult to calculate the Hessian (V''_N) , by adding the expression $p \times I$ to the Hessian.⁶ The second advantage is that, in a case where some unknown parameters cannot be accurately estimated, regularisation enables the posterior values to be pulled toward their priors. The influence of the prior is stronger for those parameters that have a minor effect on V_N , i.e. those which, without regularisation, would less contribute to the good fit of the model.

In the previous part we analysed the criterion function, determined by means of the prediction error. In this paper, the criterion is defined using the likelihood function. In a case like this, the problem can be defined as:

$$\hat{\Theta}_{N}^{MLE} = \arg \max_{\theta} \log L(\theta; Y)$$

where $\log L(\theta, Y)$ is the logarithm of likelihood function which needs to be maximised with respect to the vector of parameters θ . As already mentioned, regularisation will modify the objective function, which will result in a transformation of the aforementioned problem that can be written, in the case of regularisation, as follows:

$$\hat{\theta}_{N}^{p} = \arg \max_{\theta} \log L(\theta; Y) - p \sum_{i} \frac{(\theta - \overline{\theta}_{i})^{2}}{\sigma_{\overline{\theta}_{i}}^{2}}$$

We can see that, along with the likelihood function, the equation includes another expression which is an equivalent to the function $l(\theta)$, multiplied by the penalising term. Here, a truncated normal distribution is a priori assumed, so that $\overline{\theta}_i$ and $\frac{1}{p}\sigma_{\overline{\theta}_i}^2$ can be interpreted as the mode and variance of the prior distribution which has its lower and upper limits. The unknown parameters, estimated by maximising the said function are parameters of the posterior distribution; the estimates of these parameters are shown in Annex 1. At this point,

⁶ For the proof, see Ljung et al. (1992).

it should be noted that the estimated posterior parameters, as well, are always within the specified limits (Θ_i^{υ} and Θ_i^{L}). As before, term *p* represents penalisation, and it is clearly shown that, for any given $\sigma_{\overline{\Theta}}^2$, the higher the value of *p* the "stronger" the *a priori* information. In this paper, p = 1, which enables the a priori dispersions⁷ to be interpreted as standard deviations.

It should be emphasized that the results for the analysed sample will, in addition to the mean estimates of unobserved variables, also contain confidence intervals obtained analytically, by treating the model and estimated parameters as a real data-generating process. In that case, the stated intervals involve uncertainty of the estimated unobserved variables.

In the Kalman filter, the set of observed variables comprises output, the annual rate of core inflation, unemployment rate, annual rate of change in industrial production and retail trade and the current account balance. The model is estimated for the period from the first quarter of 2000 to the fourth quarter of 2010, for the said observed variables, except for inflation expectations.

The inflation expectation time series was taken from the *Consensus Economics Forecast*, available for Croatia only since 2006, whereas older data were estimated through the model as unobserved data. Real gross domestic product (GDP, at the previous year's prices, reference year 2000, in kuna) was used as a measure of output. The measure of unemployment was the administrative unemployment rate, whereas industrial production and retail trade were measured by the base index of the total volume of industrial production and base retail trade index respectively.⁸

The annual inflation rate was calculated using the core consumer price index, whereas the share of the current account deficit in GDP was used as the relevant current account deficit series, with both variables expressed in nominal terms and in domestic currency, but, in order to partially smooth volatility, annual moving averages were used.

All the series used were taken from the CBS (Croatian Bureau of Statistics) in an unadjusted form, except for the current account deficit, which was taken from the CNB (Croatian National Bank) statistics, and the aforementioned expected inflation.⁹ In order to eliminate seasonal effects, all the used variables that incorporated seasonal components were seasonally adjusted, using the CNB's seasonal adjustment procedure based on ARIMA X-12.

In order to avoid excessive deviation of the rate of change in potential output from its steady state value, the model was expanded by an additional measurement equation, which represents our prior belief about the degree of volatility of potential output. This prior belief is expressed by the relationship between the change of potential output and its steady state value:

$$4 \cdot (\log \overline{Y}_t - \log \overline{Y}_{t-1}) = G_{ss}^{\overline{Y}} + \varepsilon_t^{ME}$$
⁽²⁰⁾

In this case a high value of the standard deviation ε_i^{ME} would reflect a prior belief that the potential growth rate deviates sharply from its steady state value. When estimating the model, the prior for the standard deviation is set as the value of the standard deviation of actual GDP in the reference period divided by 12. The steady state rates of growth in industrial production and retail trade are calibrated to three percentage points, whereas other parameters are estimated. The prior values of parameters and standard deviations, together with estimated posteriors are shown in Annex 1.

⁷ Second column in Annex 1.

⁸ The base year for both series is 2005.

⁹ The CBS publishes all these series (except the GDP series) on a monthly basis. Therefore, the necessary quarterly series are calculated as average monthly values for particular quarters.

4 Results

4.1 Potential output, output gap and inflation

Annex 3 shows a mean estimate of potential output, together with the confidence interval of two standard deviations and the actual (realised) output. The period shown is from the first quarter of 2000 to the fourth quarter of 2010, including the model forecast for the period from the first quarter of 2011 to the fourth quarter of 2012.

Expectedly, the potential output series is relatively smooth, showing positive annual growth rates from the very beginning of the reference period, which started slowing down in mid-2006, so that the fourth quarter of 2009 saw the first negative annual rate of change in potential output (Annex 4). The turning point occurred four quarters after the first negative annual rate of change in actual GDP, which proved that this was not just a typical short-term deviation of output from its long-term level, but also a decrease in the long-term level of output. The annual rate of change in potential GDP was negative for one year, and it was only in the last quarter of 2010, that this rate became slightly positive again. Based on a model forecast, positive annual rates are expected to continue until the end of the forecast period.

In accordance with the mean estimates obtained, the observed output gap series can be divided into three sub-periods, two of which are sub-periods of a negative output gap and one, considerably longer sub-period, of a positive output gap (Annex 5). The first period of a negative output gap lasted ten quarters, i.e. until the second quarter of 2002, and was followed by 27 positive quarters.¹⁰ The final sub-period with actual lower than potential output lasted for seven quarters. According to a model forecast, this negative gap, to be gradually reduced, will last for another six quarters, and a positive, but negligible output gap is only expected in late 2012.

Concerning the degree of deviation, we conclude that the widest positive gap of about 6% was recorded in the first two quarters of 2008. This overheating period also saw maximum levels of actual GDP, followed by a continuous fall in GDP. As early as three quarters after the described maximum gap, the gap sign changed, as a result of a slump in actual output. The negative sign was most pronounced in the period between the first and the second quarter of 2010, when the output gap was as large as–3.5%, which was because potential output again showed slight positive rates of change, accompanied by a further fall in actual GDP.

The key feature of the specified model is the mutual relationship between inflation and potential output. Annex 5 gives a comparative view of the movements in inflation rate, measured by the annual rate of change in the core consumer price index and by the output gap in the period between the first quarter of 2000 and fourth quarter of 2012. We can easily notice that the movement of inflation with respect to the estimated movements in potential output is as earlier expected given the assumptions incorporated in the model. During the reference period, core inflation declined in times of a negative output gap and rose during periods of a positive output gap. Thus, around the periods of the minimum and maximum output gap values, core inflation also reached its minimum (second quarter of 2010), and maximum levels (third quarter of 2008). An exception in this correlation can be seen in the period from the second quarter of 2006 to the second quarter of 2007, which may be partly accounted for by the (dominating) influence of foreign prices in that period.

Annex 5 also shows that the core inflation rate adjustment lagged behind the output gap in periods of emergence from recession/expansion, which is implied by the inflation equation in the model, and is caused by the effect of the adjustment speed limitation due to restricted production capacities in some sectors. The labour market rigidity is even more obvious and is primarily reflected in the lagging of the change in the unemployment rate and unemployment rate gap behind output and output gap (Annexes 5, 6 and 7). It is also

¹⁰ This cycle was due to a recession that started in 1998. However, given the lack of data, the GDP series was used which began as late as 2000.

shown that, in both cases, a change in the sign of the unemployment rate gap occurs four quarters after a change in the output gap sign.

4.2 Effect of changes in output gap and potential output on changes in actual output

As clearly shown in Annex 4, there is a correlation between the annual growth rates of actual and potential output. Moreover, the Annex suggests the relative smoothness of the potential output growth rate, implying that the output gap is more important for short-term output movements relative to its potential. In order to confirm this conclusion, which also represents the conventional view for highly industrialised countries, we calculate the ratio between the variance of output gap changes and variance of actual output changes over horizons from one to twenty quarters.¹¹ If changes in output gap dominate the short-term changes in output, while changes in potential output dominate the long-term trend, then it can be expected that the ratio between the said variances will be above 0.5 in the first quarter and will decline thereafter. This is confirmed in Annex 8, which is a graphical representation of the ratio between the two variances.

4.3 Relative robustness of a multivariate filter

After the presentation and interpretation of results in the form of estimated values, the open issue that remains is the valuation of the obtained results and a comparison between the potential output estimate made by means of a multivariate filter and the estimate made using some other method. In such a comparison there is no formal criterion of fit for the potential output or output gap, because we are dealing with estimated values of unobserved variables. However, it is possible to define a criterion for measuring revisions of estimated unobserved series in real time, necessitated by the release of new data, as an appropriate criterion for comparing different methods. This is a way to measure and compare the stability of estimated series with respect to the change in the sample used, so that the method requiring relatively less data revision is considered relatively robust.

The analysis presented in this part of the paper is based on an output gap revision, where the HP filter¹² is used as an appropriate benchmark method for comparing the obtained results. The procedure is designed in such a manner that the appropriate unobserved series for the sample from the first quarter of 2000 to the second quarter of 2003 are estimated in the first step. In the second step, we calculate the difference between the estimated value of output gap for the last quarter included in the sample (estimated on the previously defined sample) and the value for the appropriate quarter, obtained on the entire sample. This process continues iteratively for each following quarter until the first quarter of 2008, after which a mean absolute error (MAE), i.e., the average of the absolute values of revisions, is calculated. The procedure was used in both methods, i.e. the multivariate filter and the HP filter approaches.

The results (Annexes 2, 9 and 10) clearly show that, in the said period, the output gap had smaller revisions on average for the multivariate filter than for the HP filter, suggesting that the estimated values for the multivariate filter are more robust.

We also calculated the average of the absolute values of output gap revisions for other samples. Thus, for example, Annex 11 shows movements in the mean absolute values of revisions, calculated on samples that are iteratively expanded by including additional quarters, where the largest sample comprises the period from the beginning of 2003 to the last quarter of 2010. Here, it can also be concluded that the revision is smaller on average for the multivariate filter than for the HP filter for all samples used. After the first presented period (the first quarter of 2008), average revisions for both methods increase constantly until the first quarter of

¹¹ The ratio between the rate of change variances is defined as follows: $\frac{var(y_t - y_{t-n})}{var(\log(X_t) - \log(Y_{t-n}))}$, for n=1,2,...,20.

¹² The reason for choosing the HP filter was its wide use in both the scientific and central banking communities.

2010, and decline again thereafter. The figure in Annex 11 also shows that the difference in average revisions between the two methods used increased till end-2009, but remained approximately constant thereafter.

5 Conclusion

This paper estimates the potential output of the Republic of Croatia, using a combination of the regularised maximum likelihood method and a multivariate Kalman filter based on a dynamic macroeconomic model. Apart from providing information on output itself, the multivariate filter also takes into account data on other economically relevant series, such as retail trade, industrial production index and current account deficit. Ignoring these series, as is the case when univariate filters are used, can lead to a bias in potential output estimation. The MV filter is based on a small dynamic macroeconomic model in which potential output is defined as the level of output that can be sustained in the long run, without creating a tendency for inflation to either rise or fall.

The estimated output gap series substantially follows movements in the annual inflation rate. For example, in the period of high inflation rates (2008), the output gap also reached its maximum level, but a slump in the output gap that followed led to a fall in the inflation rate. The recent economic slowdown, perceived in all the observed series, expectedly affected potential output as well. This period was not only characterized by a decline in actual output, but also, as suggested by the results of the model, by a fall in the level of potential output itself and a sharper decline in actual output than in potential output.

In the absence of a formal measure of fit for unobserved time series, the relative robustness of the output gap, estimated using the MV filter was compared to that of the output gap estimated by means of a univariate HP filter. The analysis shows that the output gap estimate, obtained by means of a multivariate filter is a relatively more robust series and that the multivariate filter should therefore be considered superior in the current analysis.

The estimated series suggests the existence of a certain degree of variability of potential output. Moreover, it is confirmed that the short-term movements in actual output are determined by output gap movements, while its long-run trend is dominated by movements in potential output.

Although the model takes into account foreign exchange by including the current account deficit in its structure, a small open economy like Croatia requires this model to be expanded with an explicitly modelled export sector. Furthermore, the prediction capacities of the model should be examined more closely. This will be the subject of our future analysis.

6 References

Baxter, M., and R. G. King (1999): *Measuring Business Cycles: Approximate Band-Pass Filters for Economic Time Series*, The Review of Economics and Statistics, MIT Press, Vol. 81(4), pp. 575-593.

Benes, J., K. Clinton, R. Garcia-Saltos, M. Johnson, D. Laxton, P. Manchev, and T. Matheson (2010): *Esti*mating Potential Output with a Multivariate Filter, IMF WP/10/285.

Benes, J., and P. N'Diaye (2004): A Multivariate Filter for Measuring Potential Output and the NAIRU: Application to The Czech Republic, IMF WP/04/45.

El-Ganainy, A., and A. Weber (2010): *Estimates of the Output Gap in Armenia with Applications to Monetary and Fiscal Policy*, IMF WP/10/197.

Hamilton, J. D. (1994): Time Series Analysis, New Jersey, Princeton University Press.

Harvey, A. C. (1989): Forecasting and Structural Time Series Models and the Kalman Filter, Cambridge University Press.

Hodrick, R. J., and E. R. Prescott (1981): *Post-War U.S. Business Cycles: An Empirical Investigation*, Discussion Papers 451, Northwestern University, Center for Mathematical Studies in Economics and Management Science.

Laxton, D., and R. Tetlow (1992): A Simple Multivariate Filter for the Measurement of Potential Output, Technical Report, No. 59, Bank of Canada.

Ljung, L. (1999): System Identification: Theory for the User, Prentice-Hall: New Jersey.

Ljung, L., T. McKelvey, and J. Sjoberg (1992): On the Use of Regularization in System Identification, 12 IFAC World Congress, Sydney, Australia, Vol. 7, June 1993, pp. 381-386.

Okun, A. M. (1962): *Potential GDP: Its Measurement and Significance*, in Proceedings of the Business and Economics Statistics Section, American Statistical Association, pp. 98-103.

Vrbanc, I. (2006): *Estimate of Potential Gross Domestic Product Using the Production Function Method*, the paper presented at the Croatian National Bank's 12th Dubrovnik Economic Conference, Dubrovnik, 28 June – 1 July 2006, http://www.hnb.hr/dub-konf/12-konferencija/mladi/vrbanc.pdf.

Annex

Annex 1 Model parameters

Parameter	Prior		Posterior	
	Mode	Dispersion	Mode	Dispersion
$G^{\overline{Y}}_{ss}$	3.000	0.300	3.000	0.036
U^{ss}	14.000	1.000	14.006	0.121
θ	0.500	0.030	0.500	0.004
α	0.500	0.050	0.500	0.006
β	0.400	0.100	0.393	0.012
Ω	0.500	0.300	0.490	0.041
ρ_1	0.800	0.150	0.800	0.018
$\mathbf{\Phi}_1$	0.800	0.150	0.801	0.017
ϕ_2	0.300	0.150	0.296	0.017
τ	0.100	0.150	0.104	0.025
ω	3.000	1.500	2.997	0.149
ρ_2	5.000	3.000	5.014	0.309
λ	2.000	3.000	1.949	0.551
σ_{ϵ^y}	1.000	0.300	3.323	0.036
$\sigma_{\epsilon_{\iota}^{\overline{G^{Y}}}}$	1.000	0.300	3.423	0.037
σ_{ϵ^u}	0.500	0.300	1.357	0.036
$\sigma_{\epsilon_l^{\overline{u}}}$	0.100	0.150	0.324	0.026
$\sigma_{\epsilon_l^{\overline{G^U}}}$	0.100	0.150	0.325	0.016
$\sigma_{\epsilon^{\pi_4}}$	0.500	0.300	1.577	0.037
$\sigma_{\epsilon^{\pi 4LTE}}$	0.300	0.300	0.720	0.039
$\sigma_{\epsilon_l^{\overline{r}}}$	0.250	0.150	0.784	0.018
$\kappa_{_1}$	0.100	0.200	0.094	0.025
κ_{2}	1.500	0.500	1.482	0.055
а	0.100	0.050	0.100	0.006
$\sigma_{\epsilon^{indpr}}$	0.400	0.100	1.414	0.012
$\sigma_{\epsilon_{l}^{\overline{INDPR}}}$	0.500	0.100	1.765	0.012
κ_{3}	0.100	0.200	0.095	0.031
κ_{4}	1.500	0.500	1.492	0.065
c	0.100	0.050	0.097	0.006
$\sigma_{\epsilon^{mm}}$	0.400	0.100	1.447	0.012
$\sigma_{\epsilon_l^{\overline{NM}}}$	0.400	0.100	1.500	0.012
κ_{5}	0.100	0.200	0.097	0.024
κ_{6}	1.500	0.500	1.433	0.079
d	0.100	0.050	0.100	0.006
$\sigma_{\epsilon^{dtr}}$	0.600	0.100	2.099	0.012
$\sigma_{\epsilon^{\overline{DTR}}_{t}}$	0.400	0.300	1.331	0.036

Annex 2 Average of the absolute values of revisions (estimated revisions for all sample periods from the first quarter of 2003 to the first quarter of 2008)

Variable	Average of absolute values of revisions	
y_t (multivariate filter)	1.193	
y_t (HP filter)	1.362	











Annex 8 Variance of the rates of change in output gap relative to variance of actual









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Additional information, such as acknowledgments, should be incorporate in the text at the end of the introductory section.

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A single line spacing and A4 paper size should be used. The text must not be formatted, apart from applying bold and italic script to certain parts of the text. Titles must be numerated and separated from the text by double-line spacing, without formatting.

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