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Behavioural Model of Assessment of Probability of Default and the Rating of Non-Financial Corporations

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Abstract

Basel II regulations, which are also incorporated in the Basel III regulatory framework, introduced standards and guidelines for banking risk management. Credit institutions are now free to select one approach, out of the three defined, for the assessment of their risk exposure, focusing on credit risk. If a credit institution has sufficient financial resources, human resources and know-how, it will not rely on the Standardised Approach, which includes regulatory prescribed risk factor measures, but on the Internal Ratings Based Approach (IRB), which requires the institution to meet a number of criteria and prove to the regulatory authority that the internal assessments are adequate and applied in daily operations. The key risk factor under the IRB approach is the probability of default (PD), which is assessed by PD predictive models. The Croatian National Bank has developed a PD model, used for assessment of risk in the non-financial corporations sector, both on the system level and on the level of individual credit institutions, in conditions of high risk concentrations and in stress testing.

This research shows the process in which the PD model was developed and proves that such a model has greater discriminatory and predictive power with behavioural variables than without them. A special emphasis was put on the methodological approach, with its key aspects aligned with Basel II and Basel III regulations, which made significant improvements in the target characteristics of the model: its predictability and discriminatory power.

Keywords:

IRB, probability of default (PD), rating scale, non-financial corporations, Basel III, behavioural variables, application variables, model, logistic regression, information value, weight of evidence (WoE), discriminatory power, Lorenz (CAP) curve, Gini coefficient, ROC curve, binomial test, calibration, validation, stability

JEL:

G32, G02, C51

The views in this paper are those of the authors. They do not necessarily reflect the views of the Croatian National Bank or European Central Bank. All errors are the responsibility of the authors.

Contents

1 Introduction

Probability of default (PD) is one of the most important measures of credit risk under Basel III regulations (Regulation 575/2013), used in advanced approaches (IRB) for the calculation of expected loss (EL) and risk-weighted assets (RWA). The assessment of the probability of default is usually based on the financial and qualitative indicators of enterprises' operations or on the financial, property and sociodemographic characteristics of natural persons. These data are often termed application data, because they are the only data that a credit institution has at its disposal for analysis at the time of the submission of a credit application, if the client submitting the application has not yet been in a debt relationship with it. However, if the client is or has recently been in a debt relationship with a credit institution (authorised overdrafts, loans, guarantees and similar products), the credit institution also has at its disposal data on the client's behaviour in their business relationship, behavioural data, as they are called. This paper uses the logistic regression method to select, from the set of available indicators, a limited set of indicators that best discriminate obligors according to their default status and are included in the PD assessment model. This research is limited to the assessment of the probability of default for the sector of non-financial corporations.

The purpose of this research is to improve existing CNB models for the internal risk assessments of the loan portfolio of credit institutions in the Republic of Croatia under regular operating conditions. In addition to application data, which are most frequently used in practice and mainly comprise financial indicators derived from the annual financial reports of entrepreneurs (*GFI-POD*), the risk assessment also includes data from the client's business relationships with credit institutions (behavioural data), which additionally point to corporations' risk behaviour patterns that increase the probability of their default. This adds more sensitivity and dynamics to the PD function. PD predictive models are also convenient for stress scenarios because they assess the impact of financial shocks on the credit portfolio quality for the sector of non-financial corporations simulating an increase in the PD of enterprises by changing the calibration curve or by directly "shocking" input model variables for the selected segments of the loan portfolio. The results of the quantitative validation of previous models also point to the need to redesign the PD model in accordance with Basel III regulations and the best business practice. The development of the PD predictive model relies on the definition of default under Basel III regulations, with at least five years of historical data for the sample used as a basis for the development and validation of the model. The initial validation of the model on out-of-sample (OOS) and/or out-of-time data (OOT) is an additional check to prove that the model is unbiased and that it is not over-adjusted to the data used for the estimate of regression parameters.

The definition and assessment of the PD model in this research was followed by the development of a rating scale, which was also aligned with Basel III regulations. The rating scale classifies performing and non-performing enterprises, as well as associated placements, that is, exposures, into rating grades, enabling a continuous *a priori* monitoring of changes in the risk posed by individual obligors and in the credit portfolio quality of the sector of non-financial corporations, in contrast with the previous classification of obligors into A, A90, B1 – B3 and C categories, which provided only for *ex ante* monitoring of non-performing placements.

2 Literature overview

The first PD model for the assessment of enterprise credit risk was developed at the Croatian National Bank (Ivičić and Cerovac) in 2009. The model included financial indicators derived from the annual financial reports of entrepreneurs. The results of the multivariate logistic regression showed that forecasting the probability of default[1] was primarily influenced by the equity to total asset ratio and the ratio of earnings before interest and taxes (EBIT) to total liabilities, which were negatively correlated with the probability of default. In addition to the mentioned indicators, forecasting probability of default was also strongly influenced by liquidity, profitability and sales indicators as well as by construction and real estate sector affiliation. Given the then availability of data collected from credit institutions within regulatory and statistical reporting to the Croatian National Bank pursuant to the Decision on the classification of placements and contingent liabilities of banks (2003) in effect at the time, data on credit institutions' exposures to individual business entities were collected only for the portfolio of large loans, with placements to one obligor exceeding HRK 200,000, 300,000, 500,000 and 700,000, depending on the total amount of credit institution assets. The new Decision on the classification of placements and contingent liabilities of credit institutions, which came into effect in 2009, redefined the thresholds for individually significant exposures, setting them at HRK 500,000, 1,000,000 and 1,500,000, depending on the total amount of credit institution assets. Owing to these data limitations, regression parameters could be estimated only for the portfolio of a credit institution's individually significant exposures before the crisis.

Nehrbecka (2015) analysed PD for non-financial corporations with total exposures of at least EUR 1.5m in the period between 2007 and 2012. In addition to financial indicators and dummy variables, some behavioural variables were also used, such as the number of bank-enterprise relationships, the share of interest due in total exposure, the share of national currency-denominated loans in total exposure and the share of open credit lines in total exposure (4-month and 6-month medians). The parameters of the model were estimated with transformed input variables using the weight of evidence (WoE) method, and the criteria applied in the univariate selection of independent variables included their information value (IV) and the Gini coefficient. The model's discriminatory power was tested by means of the Gini coefficient and the Kolmogorov-Smirnov test (K-S test).

Flores et al. (2010) described the use of behavioural variables in the PD model for the ten largest credit card institutions. The described model is specific in that it applies only behavioural variables, two of which are based on default in previous periods.

Pursuant to Basel regulations, PD assessments have to be validated at least annually on out-of-sample (OOS) data and/or new, out-of-time (OOT) data. Quantitative tests include tests for the rating system's predictive power, efficiency, calibration and stability. Quantitative methods used for these tests differ from one author to another. In a collection of papers edited by Engelmann and Rauhmeier (2011) several authors recommend the CAP curve, Gini (AR) coefficient, ROC curve, AUROC coefficient and Brier score for the testing of discriminatory power, and the binomial test, Hosmer-Lemeshow test and Spiegelhalter test for the testing of calibration. Tasche (2006) refers to the CAP curve and the associated Gini coefficient, ROC and AUROC, the Pietra coefficient, CIER, information value, Brier score and other tests as discriminatory power tests, and to the binomial test, Hosmer-Lemeshow test (χ^2-test), normality test and "traffic lights" as calibration tests. Medena et al. (2006) use some of these tests, and Baesens (2016) also describes the criteria for assessing the discriminatory power of the model by means of the Gini coefficient.

1 The default criteria used in the cited research are slightly different from the default establishment criteria adjusted with the Basel guidelines and directive.

3 Data

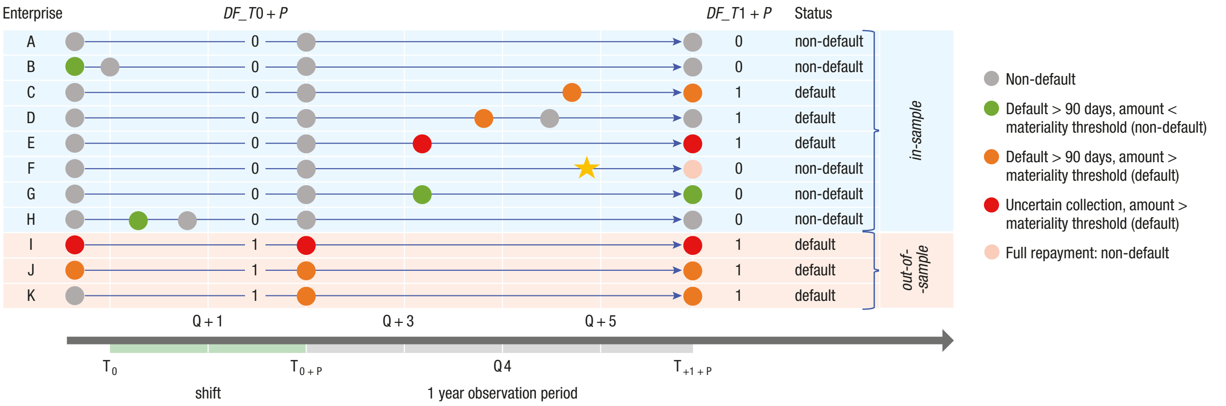
The development sample for the estimate of regression parameters comprises data on the non-financial corporations (ESA 2010[2]) to which credit institutions were exposed in the 2011 to 2015 period. The estimate of regression parameters requires data from *GFI-POD* annual financial reports for the business years immediately preceding the beginning of the observation period (T_0 2011– 2014) for all enterprises in the sample, and information on changes in the default status from 1 January to 31 December in the year of the observation period T_{+1}, 2012 – 2015. The out-of-time (OOT) validation sample comprises the non-financial corporations to which credit institutions were exposed in 2015, with the known outcomes of default in 2016, with risk parameters based on 2015 *GFI-POD* reports. Both samples exclude enterprises defaulting in a given moment during the year T_0, which precedes the observation period (monthly data frequency). This allows for the inclusion in the sample of only those enterprises for which the necessary data are available and to which credit institutions are exposed, an additional criterion being that these enterprises regularly met their commitments before the beginning of the observation period (were not in default in the year T_0), because the probability of default is assessed (Figure 1).

The observation period for the one-year PD is always on the one-year horizon, but the beginning of the observation period need not coincide with the calendar start of the year. Depending on the shift of the beginning of the observation period in relation to the date of the annual financial reports of entrepreneurs, the periods observed related to the 0-month shift: 31 Dec. T_0 – 31 Dec. T_{+1}, the 3-month shift: 31 Mar. T_{+1} – 31 Mar T_{+2}, and the 6-month shift: 30 Jun. T_{+1} – 30 Jun. T_{+2}, where T_0 is the year to which a financial report refers, and indices with the years +1 and +2 stand for the number of years elapsed since the year of the financial report (Figure 1). The shift can be used to bridge the time gap between the date of the financial report and the date of its public release, which in practice approximately lasts between four and six months, which extends the validity deadline for the calculated rating. This may slightly weaken the model's predictive properties due to the increased "obsolescence" of the financial reports included in the model.

The final estimate of regression parameters was made based on the 0-month shift, with the result that the sample comprises the one-year observation period 31 Dec. T_0 – 31 Dec. T_{+1}. The period for the training sample was chosen according to the availability of consistent credit institutions' data on individual exposures to the sector of non-financial corporations.

The initial sample for the training of the model contains 144 variables, of which 79 are application

Figure 1 Creation of the development sample



Source: Author's calculation.

2 Enterprises broken down by sector according to the European System of Accounts, ESA 2010.

variables (financial indicators based on annual financial reports) and 65 are behavioural variables (variables derived from data collected from credit institutions' regulatory and statistical reports). The training sample consist of 69,049 observations (non-financial corporations for the period between 2011 and 2014). The out-of-time (OOT) validation sample comprises 17,455 non-financial corporations with annual financial reports for 2015 and the outcome of default in 2016.

The development of statistical predictive models is a statistically based process of selecting risk factors, that is, independent model variables that are the best predictors of the probability of occurrence of the modelled event, i.e., the dependent variable, in this case the binary variable of default (the variable with two possible states : 0 – non-default, 1 – default). The best business practice implies the preparation of a sample that includes all potentially predictive independent variables for each enterprise in the sample and a known outcome at the end of the observation period, in this case lasting one year from the moment in which the probability is assessed. The reference moment is determined by the year of the financial report of the entrepreneur, that is, 31 December of the year for which the financial report was compiled.

4 Methodology

The basic assumption for the development of the PD model under Basel III regulations builds on the definition of default. The initial selection of explanatory variables is carried out by the univariate analysis, which assesses the predictive properties of each independent variable in order to exclude from further analysis all variables that do not meet the criteria of predictivity (the model's discriminatory power) and data completeness (the share of missing values of a variable in the sample should be as low as possible). The multivariate analysis excludes highly correlated variables from the sample in order to avoid the model potentially overfitting with the data on which the regression parameters are estimated. The final list of variables – candidates for the model – is composed of low correlated, highly predictive and sufficiently complete variables, which may also be previously transformed (outlier-restricted, weight of evidence-transformed, standardised or linearised by transformation functions) in order to achieve as good as possible monotonic linear dependence between the independent and dependent variables. The regression analysis includes a final set of selected variables, and the variables that remain in the final model are those that meet the estimate's conditions for economic justification (a variable is meaningful, the estimate's sign complies with the expected sign regarding the correlation of risk with the variable's value) and significance tests (p-values). The selected models are further validated, once they have been calibrated in order to enable the calculated probabilities to reflect the real probabilities of default. The calibration of the model is followed by the definition of the rating-scale. Initial validation tests are conducted on the training sample, and results are confirmed on the test sample (OOT).

The most frequently used quantitative tests (mentioned in section 2 Literature overview) for the estimate of the model's discriminatory power, which are applied in this research too, include the CAP curve and the associated Gini coefficient as the tests of the model's discriminatory power, and the binomial test for the testing of individual rating grades as the calibration test for the composition and validity assessment of the rating scale.

4.1 Definition of default

For the purpose of calculating risk-weighted assets under the IRB approach and weighting exposures in default under the standardised approach[3], the default of an obligor is considered to have occurred when either or both of the following have taken place:

3 Credit institutions calculate the amount of risk-weighted assets (RWA) for the capital requirement. They can apply the Standardised Approach (STA) with the regulatory prescribed weights for asset items or they can apply the approaches based on their own assessments of risk factors on the condition that they comply with the competent authority's criteria for the authorisation of the Internal Ratings Based approach (IRB).

a) the institution considers that the obligor is unlikely to pay its credit obligations to the institution, the parent undertaking or any of its subsidiaries in full (without recourse by the institution to actions such as realising security);
b) the obligor is more than 90 days past due on any material credit obligation to the institution, the parent undertaking or any of its subsidiaries.

The materiality of a credit obligation past due (for the purpose of item (a)) is estimated against the threshold defined by the competent authorities. The threshold must reflect a level of risk that the competent authority considers appropriate. The European Banking Authority (EBA) has drafted the Regulatory Technical Standards (RTS), which specify the conditions for setting the materiality threshold of a credit obligation past due by the competent national authority, and the guidelines on the default of an obligor.

With an aim of achieving the fullest possible harmonisation with the regulatory definition of default and the accepted materiality threshold (adopted in line with the Regulatory Technical Standard concerned) to be applied by institutions in the Republic of Croatia for the calculation of the amounts of risk-weighted exposures to companies, the selected definition of default includes two components:

a) 90 days past due on a material credit obligation exceeding the determined materiality threshold according to monthly data on overdue claims; and
b) uncertainty of collection, identified by the formation, in at least one credit institution, of specific value adjustments for the amounts exceeding the materiality threshold or by the fact that an obligor is past due on any material credit obligation (risk categories A90, B1, B2, B3 or C).

The materiality threshold is defined in the absolute amount of HRK 3,750. The analysis also considered HRK 1,750 and HRK 5,000. The prescribed relative component (2.5% of the total exposure to an obligor) was not included in the definition of default primarily because of the supervisory practice that recognises each specific value adjustment as default.

4.2 Weight of evidence (WoE) and information value (IV)

The logistic regression implies a monotonically increasing (or decreasing) function of the independent variable. In some cases the condition of the risk function's monotonic growth is not fulfilled, with the result that regression errors during the growth interruption periods are greater: the left graph in Figure 2 shows a significant deviation of the regression line *Linear (DR)* from the realised default rate (DR) in relation to the right graph with small deviations from the realised DR, and the estimate of the regression line *Linear (DR)* on WoE transformed values of the default rate (DR). In such cases, independent variables are most often transformed by the weight of evidence (WoE) in order to achieve the monotonic risk function on the value of the transformed variable (Figure 2) and in this way increase its predictive power (information value).

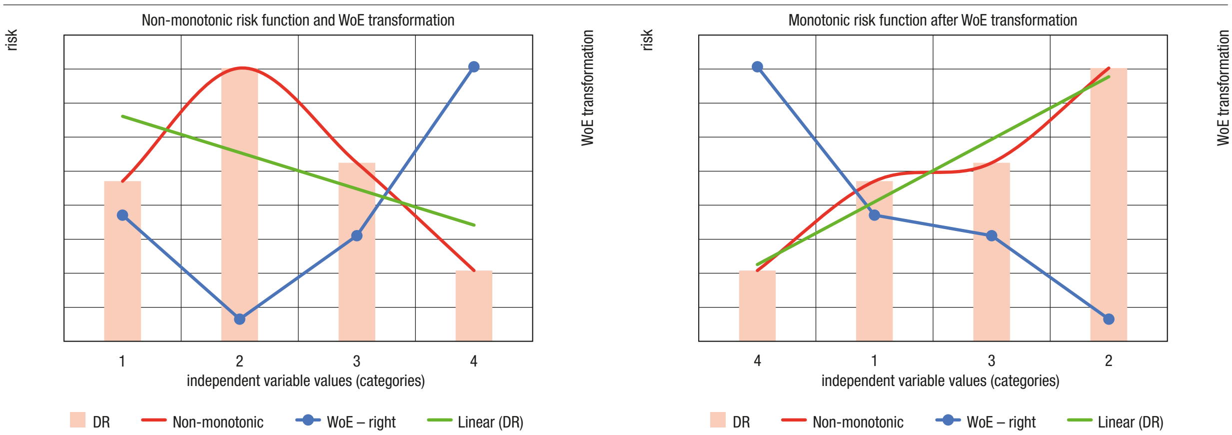
The weight of evidence transformation (*WoE*[4]) is a kind of transformation that relates the predictive power of the independent variable's value to the dependent variable. The WoE calculation is based on the classification of the values of the input variable into categories by maximising the information value of each category and, in turn, the difference between the categories (the supervised discretization method), with the transformed WoE values and information value calculated for each category *i* and in aggregate for all variable values:

$$WoE_i = \ln\left(\frac{\%\text{ “good”}}{\%\text{ “bad”}}\right) \tag{1}$$

The information value aids the selection of model variables with a greater predictive power. Based on their total information value, variables are ranked according to their predictive power; the information value is calculated as follows:

4 Siddiqi (2006).

Figure 2 WoE transformation into a monotonically increasing function



Source: Author's calculation.

$$IV_i = \sum (\% \text{"good"} - \% \text{"bad"}) \cdot WoE_i \tag{2}$$

According to Siddiqua (2006), the information values of variables that may be used in modelling because they contain a sufficient level of information are those values exceeding 0.1 (medium power, Table 1).

Table 1 Predictive power with regard to information value

Information value	Predictive power
< 0.02	unsuitable for modelling
0.02 – 0.1	weak power
0.1 – 0.3	medium power
0.3 – 0.5	excellent power
> 0.5	extraordinary power

Source: Siddiqi (2006).

4.3 Model parameters estimate

Default is the dependent binary variable y_i in the estimate of regression parameters, which assumes value 0 if the enterprise i is not in default or value 1 if the enterprise i is in default:

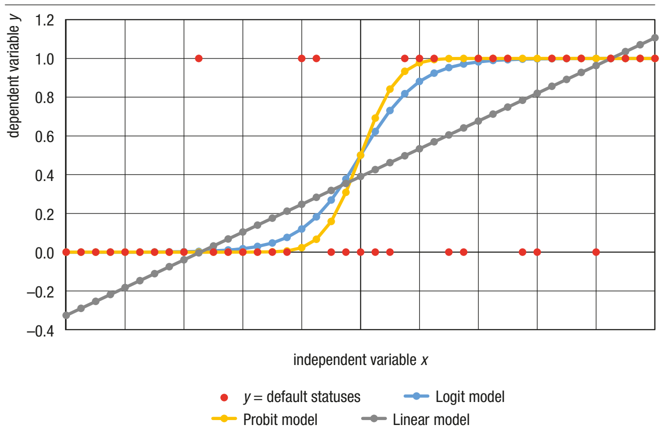
$$y_i = \begin{cases} 0 & \text{... non} - \text{default} \\ 1 & \text{... default} \end{cases} \tag{3}$$

Defaults can be estimated by means of various regression functions, such as the linear regression described by the linear equation (which is inadequate if the dependent variable is binary because the linear function is not limited to the 0 – 1 range) and the most often applied regressions for binary dependent variables: the logistic regression (logit), which uses the logarithm transformation, and the probit regression, which uses the normal cumulative distribution function[5] (Figure 3).

The probability of default function is estimated by the multivariate logistic regression (logit). The logistic regression has several advantages over other types of regressions: the result of the logistic regression can be directly identified with the probability of default, and it also facilitates verifying the economic meaningfulness of interdependence between the estimated risk probability and the independent variable. The multivariate logistic

5 For details on the described regressions and interpretations, see Hyden and Porath (2011).

Figure 3 Regression functions



Source: Author's presentation.

regression estimates the coefficients of the vector β' starting from the assumption of linear regression (4) and its non-linear transformation by the function (in this case, the logistic function (5)) in order for the achieved estimates to match the probabilities in the range of 0 to 1.

$$Score_i = \boldsymbol{\beta}' \boldsymbol{x}_i \tag{4}$$

$$PD_{i\ uncalib.} = \frac{1}{1+e^{-Score_i}} = \frac{1}{1+e^{-\beta' x_i}} \tag{5}$$

where $\beta' = (c_1, \beta_1, ..., \beta_k)$ is the the vector k + 1 of the estimated coefficients of the PD model, including the constant c1 and the coefficients β_j for the transformed application and behavioural variables (their WoE values), $\boldsymbol{x}_i = (1, x_{i1}, ..., x_{i,k})$ is the vector of the transformed application and behavioural variables, and number 1 is a vector constant.

4.4 Model's discriminatory power

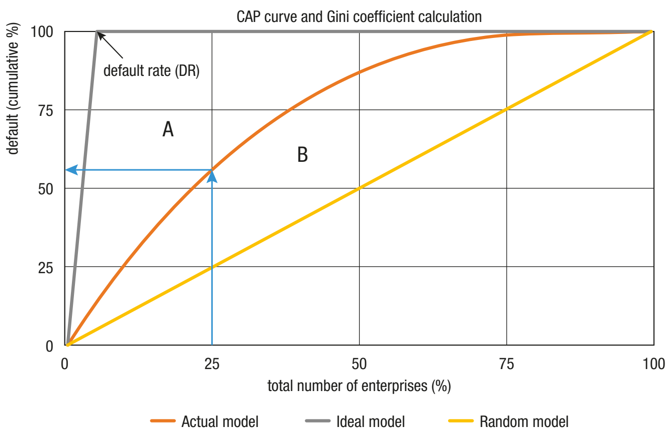
A model's discriminatory power is its ability to differentiate between "good" (non-defaulting) and "bad" (defaulting) enterprises. The most often used measures of a model's discriminatory power are the Brier score and the Lorenz (CAP) curve with the accompanying Gini coefficient. This research uses the Gini coefficient as a measure of the model's discriminatory power and the Lorenz (CAP) curve for the visualisation of the measure of discriminatory power.

Lorenz (CAP) curve and the Gini coefficient

A model's discriminatory power is most often represented by the Lorenz (CAP) curve. The CAP curve shows the percentage of "bad" enterprises (axis y, "in default") included in the percentage of total enterprises (axis x, "total number of enterprises"), with the enterprises aligned according to the values of the analysed variable in the descending order as regards risk (Figure 4). The discriminatory power of a variable (or the whole model) is higher if "bad" enterprises are more concentrated on the left side of the axis x. For example, 25% of the total number of enterprises presented below includes more than 55% of the total number of enterprises in default ("bad").

The Gini coefficient (AR) is a quantified measure of the model's discriminatory power derived from the CAP curve. It is calculated as a ratio between two areas delineated by the curves of the actual and accidental models (area B) and the ideal and accidental models (area A+B):

Figure 4 Illustration of the model's discriminatory power



Source: Author's presentation.

$$GINI = B / (A + B) \tag{6}$$

The acceptable level of the model's discriminatory power in practice implies Gini coefficient values greater than 0.4 (Table 2).

Table 2 Gini coefficient's discriminatory power

Gini coefficient (AR)	Quality
AR < 0	**no discrimination**
0 < AR < 0.4	poor discrimination
0.4 < AR < 0.6	acceptable discrimination
0.6 < AR < 0.8	excellent discrimination
0.8 < AR < 1	**exceptional discrimination**

Source: Baesens (2016).

4.5 Model calibration

The uncalibrated PD of an individual enterprise is calculated for the estimated parameters β of the PD model of each segment in the manner described by expressions (4) and (5) in section 4.3.

The relative frequency of default *RDF* is defined as the ratio between the number of defaulting ("bad") enterprises and the number of non-defaulting ("good") enterprises, expression (7), while the connection between the relative frequency and probability of default is given in expression (8).

$$RDF = \frac{\text{number of “bad”}}{\text{number of “good”}} \tag{7}$$

$$RDF = \frac{PD}{1 - PD} \tag{8}$$

Source: OeNB and FMA (2004).

The adjustment (calibration) of the relative frequency of default of the sample to the relative frequency for the population (central tendency) is defined by the following expression:

$$RDF_{calibrated} = RDF_{uncalibrated} \frac{RDF^{CT}}{RDF^{S}} \tag{9}$$

Source: OeNB and FMA (2004).

where:

$RDF_{calibrated}$ – calibrated relative frequency of default;

$RDF_{uncalibrated}$ – uncalibrated relative frequency of default;

RDF^{CT} – long-term average of the relative frequency of default for the population;

RDF^{S} – average relative frequency of default for the sample.

A combination of expressions (7), (8) and (9) may be used to calculate the calibration function for the adjustment of the uncalibrated model PD, PD_i of an enterprise, to the central tendency, as shown by the following formula:

$$PD_i^{CT} = \frac{PD_i \cdot (1 - DR^S) \cdot DR^{CT}}{(1 - PD_i) \cdot DR^S \cdot (1 - DR^{CT}) + PD_i \cdot (1 - DR^S) \cdot DR^{CT}} \tag{10}$$

where:

PD_i^{CT} – PD of enterprise *i* calibrated to the central tendency;

PD_i – uncalibrated model assessment of the PD of enterprise *i*;

DR^{CT} – calculated central tendency (long-term average) of the default rate;

DR^{S} – average sample default rate for the estimate of model parameters.

Binomial test

Assuming that default is independent, the binomial test is used for the assessment of the correctness of the envisaged PDs (calibration) for individual rating grades[6]. In this manner, the critical values of the number, i.e., of the rate of defaults for each rating grade is determined. The modified binomial test uses the approximation of the binomial distribution by the normal distribution according to the central marginal theorem when the number of enterprises is large enough. The minimum number of enterprises in the rating grade may be assessed by means of the conditions for the approximation of the binomial distribution by the normal distribution:

$$N_i \overline{PD_\iota}(1 - \overline{PD_\iota}) > 9 \Rightarrow N_{i\min} = \frac{9}{\overline{PD_\iota}(1 - \overline{PD_\iota})} \tag{11}$$

where N_i is the number of enterprises in the rating grade *i* with the average estimated probability of default $\overline{PD_\iota}$, and N_{imin} is the minimum number of enterprises in the rating grade *i* to which the approximation of the binomial distribution by the normal distribution applies.

The confidence level applied in the test is $\alpha = 95\%$. For each rating grade *i* of the model critical values were calculated for the selected confidence category α: Inf_i – lower and Sup_i – upper limit of the test for the realised default rates:

$$Inf_i = \overline{PD_\iota} - \Phi^{-1}(\alpha) \cdot \sqrt{\frac{\overline{PD_\iota} \cdot (1 - \overline{PD_\iota})}{N_i}} \tag{12}$$

$$Sup_i = \overline{PD_\iota} + \Phi^{-1}(\alpha) \cdot \sqrt{\frac{\overline{PD_\iota} \cdot (1 - \overline{PD_\iota})}{N_i}} \tag{13}$$

where $\overline{PD_\iota}$ is the average estimated value of the PD of the rating grade *i*, N_i is the total number of enterprises in the rating grade *i*, while $\Phi^{-1}(\alpha)$ is the inverse cumulative function of the normal distribution for the confidence level α, medium value 0 and standard deviation 1. If the observed default rate is lower than the critical value Inf_i for $\alpha = 95\%$, the rating grade ensures an additional level of conservativity (PD is overestimated), if the observed default rate is between the critical values Inf_i and Sup_i, the rating grade is adequate to the realised

6 According to Blochwitz et al. (2011).

default rates with the predetermined confidence level (PD is good probability estimator) and if the observed default rate exceeds the critical value Sup_i, the rating grade underestimates risk, which is unacceptable for the rating scale. This research applied the binomial test in defining the rating-scale.

5 Regression results

5.1 Univariate analysis

5.1.1 Application and behavioural variables

Application variables are based on quantitative data and, where available, qualitative data, on enterprises being assessed. Quantitative application variables are mostly based on the financial reports of entrepreneurs that provide a basis for the calculation of operation indicators, including the indicators of liquidity, indebtedness, activity, cost-effectiveness, profitability and investment. Qualitative application data may include the number of employees (also available from the financial reports), market share, market appearance, age of the enterprise, existence of business strategy, availability of public disclosures and data on connected persons, subjective assessment of management quality, and other available information. Application variables have a low, mostly annual frequency, which is why they are less predictive than behavioural variables. The initial application dataset included the annual financial reports from the period between 2008 and 2015, the reason being that *GFI-POD* forms for this period are consistent, that is, the forms relating to 2008 and 2009 differ only slightly from the forms for the 2010 to 2015 period, while the structure of reports for the periods preceding 2008 differs more considerably from that of more recent reports.

Behavioural variables quantitatively reflect the behaviour of enterprises, with an emphasis on their behaviour in the business relationship with the credit institution. They may result from the credit institution's transactions (repayments, withdrawals, write-offs, payment transactions), the business relationship with credit institutions (the number of credit institutions, the number of products used, exposure amounts, contracted interest rates) and the credit relationship with credit institutions (the classification of exposures, delays in the repayment of outstanding claims, etc.). Behavioural variables may also be data on frozen accounts, the lists of enterprises that do not pay salaries to their employees, enterprises about to undergo pre-bankruptcy and bankruptcy-proceedings as well as data from similar negative or positive information sources. The frequency of behavioural variables is considerably higher than that of application variables. It may be a daily as well as a decade or monthly frequency, the latter two being the most frequently available at the CNB. Because some significant behavioural variables were not systematically collected in the past and resulting data limitations, behavioural variables were restricted to the 2011 to 2014 period in the training sample, and to 2015 in the validation sample, as information on the outcomes of default in 2016 were also required.

5.1.2 Initial selection of variables based on univariate analysis results

The criteria for the initial selection of variables include the satisfactory discriminatory power of a variable, that is, of the univariate model (with only one variable) expressed by the Gini coefficient and the completeness of a variable, expressed by the share of observations that are not null in the total number of observations.

The criterion for the initial selection of variables that are potential candidates for the model in the univariate analysis is that the Gini coefficient is above 0.3 and the completeness of variables is above 80% in all analysed years. Both criteria are met by 29 variables (Table 3).

5.1.3 Winsorisation

The limitation of outliers in variables, also termed winsorisation, after Charles P. Winsor, is the process of removing extreme values in the variables from the sample, that is, of replacing extreme values by the set percentile of the variable distribution. The application of variables with unlimited values in the regression resulted

Table 3 Univariate analysis – the selected “long” list of variables

				Criteria (> from)		
				0.3	80%	
Variable description	Varijable	Total min. of Gini coefficient	Total min. of completeness	Gini coefficient	Completeness	Composite
Share of average overdue claims > 90 days in T0 + 3 months in annual average credit exposure	BH_DNP_3MF_R	0.458	91.5%	1	1	1
Share of average overdue claims > 90 days in T0 +/–3 months in annual average credit exposure	BH_DNP_3MPF_R	0.451	91.5%	1	1	1
Share of maximum overdue claims (CI-instrument) > 90 days in annual average credit exposure	BH_DNP_MAX_T0_R	0.305	92.6%	1	1	1
Share of sum of overdue claims > 90 days in T0 annual average credit exposure	BH_DNP_SUM_T0_R	0.342	92.6%	1	1	1
Share of average overdue claims up to 30 days in T0 + 3 months in average annual credit exposure	BH_DNP30_3MF_R	0.320	91.5%	1	1	1
Share of average overdue claims up to 30 days in T0 +/–3 months in average annual credit exposure	BH_DNP30_3MPF_R	0.353	91.5%	1	1	1
Share of average overdue claims up to 30 days in T0 + 6 months in average annual credit exposure	BH_DNP30_6MF_R	0.372	91.5%	1	1	1
Share of average overdue claims up to 60 days in T0 + 3 months in annual average credit exposure	BH_DNP60_3MF_R	0.529	91.5%	1	1	1
Share of average overdue claims up to 60 days u T0 +/–3 months in annual average credit exposure	BH_DNP60_3MPF_R	0.556	91.5%	1	1	1
Share of average overdue claims up to 60 days in T0 + 6 months in annual average credit exposure	BH_DNP60_6MF_R	0.657	91.5%	1	1	1
Share of average overdue claims up to 60 days in T0 – 7 months to T0 – 1 month in annual average credit exposure	BH_DNP60_71MP_R	0.390	91.2%	1	1	1
Share of maximum overdue claims (CI-instrument) up to 60 days in T0 annual average credit exposure	BH_DNP60_MAX_T0_R	0.439	92.6%	1	1	1
Share of sum of overdue claims up to 60 days in T0 annual average credit exposure	BH_DNP60_SUM_T0_R	0.444	92.6%	1	1	1
Share of average overdue claims up to 90 days in T0 + 3 months in annual average credit exposure	BH_DNP90_3MF_R	0.425	91.5%	1	1	1
Share of average overdue claims up to 90 days in T0 + 6 months in annual average credit exposure	BH_DNP90_6MF_R	0.576	91.5%	1	1	1
Share of maximum overdue claims (KI-instrument) up to 90 days in T0 in annual average credit exposure	BH_DNP90_MAX_T0_R	0.315	92.6%	1	1	1
Share of sum of overdue claims up to 90 days in T0 in annual average credit exposure	BH_DNP90_SUM_T0_R	0.316	92.6%	1	1	1
Current ratio: current assets/short-term liabilities	I1	0.354	100.0%	1	1	1
Liquidity indicator III: current assets/total assets	I10	0.309	100.0%	1	1	1
Average payment period: short-term liabilities/total expenses	I16	0.313	100.0%	1	1	1
Net liability coverage after-tax profit + amortization/(debt – reserves)/365	I27	0.347	100.0%	1	1	1

Cash flow II: after-tax profit + amortisation/debt/365	I28	0.368	100.0%	1	1	1
Debt service coverage ratio, DSCR II: EBITD/debt due to FI	I30	0.324	100.0%	1	1	1
Payables turnover: accounts payable/sales revenue	I39	0.325	100.0%	1	1	1
Net rentability of assets (ROA): after-tax profit/total assets	I57	0.310	100.0%	1	1	1
Gross rentability of assets: pre-tax profit/total assets	I58	0.315	100.0%	1	1	1
EBIT II: pre-tax profit/short-term liabilities	I67	0.333	100.0%	1	1	1
EBIT V: pre-tax profit/total liabilities	I70	0.340	100.0%	1	1	1
Profitability indicator III: income from regular operations/total liabilities	I72	0.337	100.0%	1	1	1

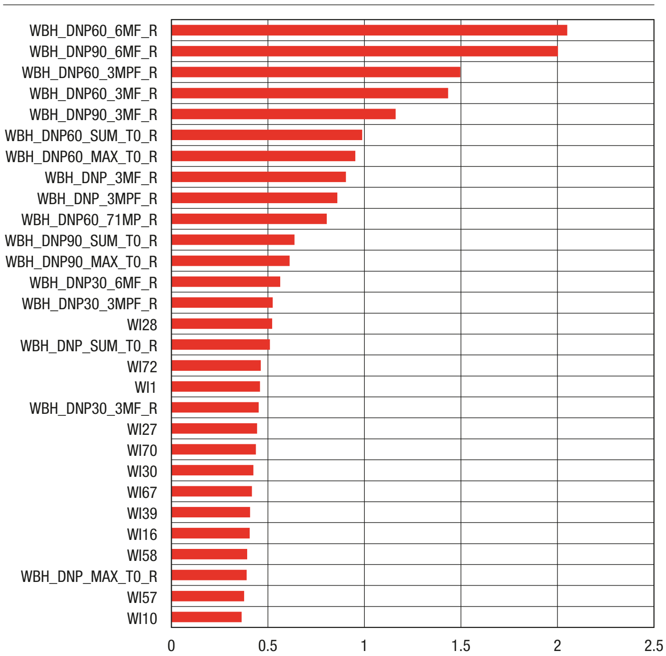
Sources: FINA, CNB and author's calculation.

in a high regression error, so that extreme values in selected application and behavioural variables were limited in distribution tails in 2% of the cases, that is, lower extremes were replaced by the 1st percentile and higher extremes by the 99th percentile of the distribution. This considerably reduces sample variability, with the result that extremely high and extremely low variable values can be included in the regression without any loss of information, which, according to Yaffee (2002), improves the robustness and representativeness of the regression (e.g. the corrected coefficient of determination and the estimated standard regression deviation). Variables with limited outliers are marked with an additional prefix "W".

5.2 Multivariate analysis

The multivariate analysis examines the correlations of selected variables and excludes highly correlated variables (except the most significant ones) from further analysis to resolve the potential problem of collinearity. The multivariate analysis used correlation matrices, while highly correlated variables were rejected if the

Figure 5 Information value of the variables in the "long" list



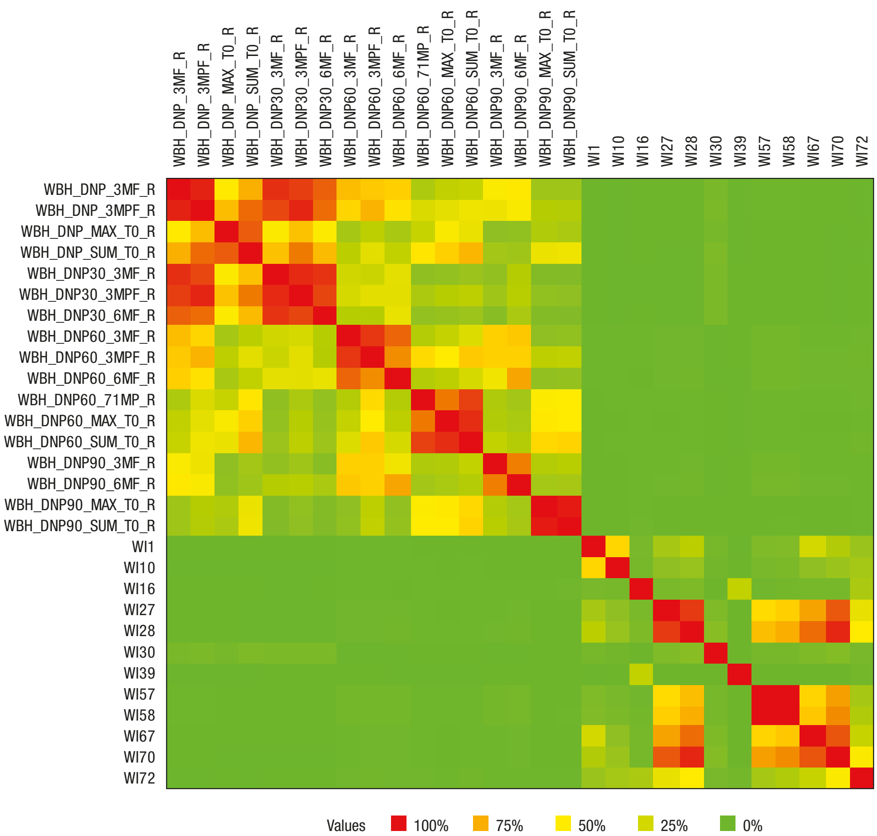
Source: Author's calculation.

correlation coefficient was higher than 60%[7]. Additional criteria for the choice of a highly correlated variable included the discriminatory power (higher power is better), completeness and information value (higher is better, Figure 5). The method of calculation of information value and its significance are described in detail in section 5.2.2, Weight of evidence (WoE) transformation of variables. Figure 5, Information value of the variables in the "long" list, shows that behavioural variables have larger information values than application variables, which also proves that they have a better predictive power (Table 1).

5.2.1 Correlation matrices

The matrices of correlations of independent variables are shown by means of a heat map, and the height of correlation for variable pairs is coded by the green-yellow-red spectrum (Figure 6). This correlation matrix shows the pairs of highly correlated variables. However, the correlation of application and behavioural variables is low, which proves that the use of both types of independent variables is justified. The final choice of variables based on the described criteria results in a set of two behavioural variables and six application variables, which will be input variables for the estimate of regression parameters.

Figure 6 Correlation heat map of variables in the "long" list



Source: Author's calculation.

Further analysis also excluded all behavioural variables that can provide early warning of default in order to avoid an autoregressive component in the model, such as the share of average overdue claims up to 60 days in $T_0 + 3$ months in the average annual credit exposure, given that non-payment until $T_0 + 4$ months can result in a delay of a material claim longer than 90 days, which is in line with the definition of default.

7 Farrar and Glauber (1967) recommend that simply correlated independent variables be limited at 80% – 90%.

Table 4 Correlation matrix of selected variables in the "short" list

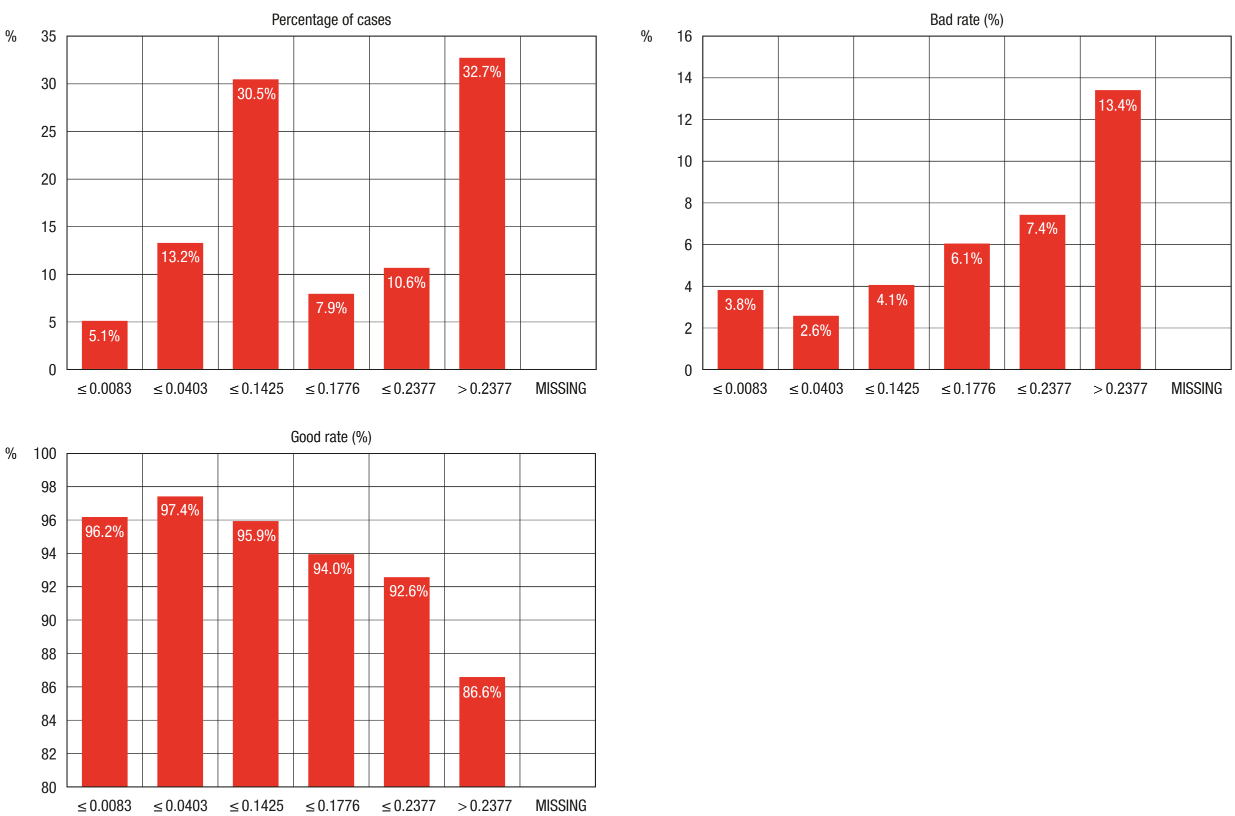
Indicator	IV	Gini coefficient	Completed	WBH_DNP60_71MP_R	WBH_DNP60_SUM_T0_R	WI1	WI16	WI27	WI30	WI39	WI72
WBH_DNP60_71MP_R	0.81	0.39	91%		53%	4%	5%	5%	2%	1%	7%
WBH_DNP60_SUM_T0_R	0.99	0.44	93%	53%		3%	6%	4%	2%	2%	6%
WI1	0.46	0.35	100%	4%	3%		11%	28%	10%	0%	24%
WI16	0.41	0.31	100%	5%	6%	11%		12%	3%	36%	30%
WI27	0.44	0.35	100%	5%	4%	28%	12%		15%	4%	45%
WI30	0.42	0.32	100%	2%	2%	10%	3%	15%		2%	11%
WI39	0.41	0.33	100%	1%	2%	0%	36%	4%	2%		9%
WI72	0.46	0.34	100%	7%	6%	24%	30%	45%	11%	9%	

Source: Author's calculation.

5.2.2 Weight of evidence (WoE) transformation of variables

All short-listed variables (with the exception of indicator variables) are weight of evidence transformed to maximise the variable information value and achieve the best possible predictive properties of the model. The

Figure 7 Distribution of WI39 variable by categories



Source: Author's calculation.

Figure 8 WoE transformed WI39 variable

Source: Author's calculation.

WI39 variable (indicator 139 with limited outliers) shows a break in the monotonic growth of the risk function (Figure 7, graph: Rate of "bad" enterprises (%)). The risk of the first category is greater than that of the second and the risk of other categories continues to grow monotonically. As a result, the WoE transformed value of the first category is lower than that of the second category and then WoE values continue to fall (a larger WoE value always signifies a lower risk of a category because it is proportional with the share of "good" enterprises and inversely proportional with the share of "bad" enterprises).

The variables transformed in this manner are input variables for the estimate of parameters of the logistic regression model.

5.3 Estimate of logistic regression model parameters

5.3.1 Segmentation and estimate of model parameters

The analysis was up to this point carried out on the complete training sample. Due to potential differences in enterprise risk depending on some quantitative factors, the estimate of logistic regression parameters should be preceded by verifying the risk profiles of some segments and deciding on the number of qualitative dummy variables in order for the model to reflect the probability of default in the best possible way with regard to the specificities of segments. The risk of segments is expressed by the default rate (DR), calculated as the share of enterprises in default in the total number of enterprises in a segment.

Table 5 shows risk profiles according to various segmentations. The segmentation of enterprises was based on sectorisation, project financing, NACE group and enterprise size (according to FINA). Private enterprises have higher risk profiles than public enterprises, project financing has a higher risk profile than other enterprises, construction (*GRAD*) has a higher risk profile than other NACE groups and medium-sized and large enterprises have higher risk profiles than small enterprises. Given the mentioned increased risk of some enterprise segments, specificities of their operation and the number of available observations in the segmented sample, especially of those in default, new dummy variables, which take the value 1 or 0, depending on the characteristic marked, are added to the sample:

a) dummy variable to mark the public enterprise sector: *DMY_SOE*

b) dummy variable to mark project financing *DMY_SOE*

c) dummy variable to mark construction activity *DMY_GRAD*.

As the difference between the risk profiles of enterprises from the segment of small enterprises is bigger than that between the risk profiles of enterprises in the segments of medium-sized and large enterprises (Table 5), the same applying to their business models, financing methods and conditions, market appearance and

Table 5 Risk profile by segments

Segmentation by variable	Segment	Number of enterprises	DR
Sector	Public enterprises	1,407	5.8%
	Majority domestic-owned private enterprises	63,096	7.5%
	Majority foreign-owned private enterprises	4,546	7.2%
Total		**69,049**	
Project financing (SPV)	Enterprises	68,958	7.4%
	Project financing (SPV)	91	18.7%
Total		**69,049**	
NACE	Construction and real estate	8,347	11.5%
	Agriculture and mining	2,070	9.3%
	Hotels and restaurants	3,915	8.7%
	Manufacturing	12,964	7.9%
	Trade	21,838	6.9%
	Transport, storage and communications	6,087	6.7%
	Service activities	11,956	5.1%
	Other activities	1,872	3.8%
Total		**69,049**	
Size	Small	64,826	7.3%
	Medium-sized	3,287	9.8%
	Large	936	9.3%
Total		**69,049**	

Source: Author's calculation.

many other business and behavioural aspects, and the number of observations and defaults is sufficient for an independent estimate of regression parameters, two models were estimated: a model for the segment of small enterprises (S) and a model for the segment of medium-sized and large enterprises (ML). The regression results suggest that significant variables for the selected segments differ, that is, that most of the variables that are significant for the small enterprise segment are not significant for the segment of medium-sized and large enterprises, which is another justification for the applied segmentation (Table 6 and Table 7).

The regression parameters β are estimated by the programme package R, function GLM for the binomial (logit) regression (binomial logistic regression).

The regression results after the removal of non-significant variables from the model are as follows[8]:

The verification of economic meaningfulness of individual independent variables *(*Table 9) and the expected sign of the coefficients of untransformed and transformed variables with regard to an increase or decrease in risk shows that regression parameters are in line with expectations[9] and applied transformations and that they meaningfully describe the risk stemming from risk factors concerned. The tables show the model variables and their assumed economic meaningfulness in the context of risk: does a high or low value of an indicator suggest a higher or lower risk level, i.e., is a variable in a positive or negative correlation with default risk (dependent variable). The realised average default rate (DR) in the training sample of the categorised input variable proves the set hypothesis of the economic meaningfulness of a variable, partially (the hypothesis is valid for most of the variable value range), or completely (the hypothesis is valid for the whole variable value range). The applied WoE transformations of input variables rectify potential breaks in the monotonic course of the function in the correlation of the independent and dependent variables and equalize the direction of

8 Table 3 contains detailed descriptions of the stated variables.

9 Ivičić and Cerovac (2009) achieved the same results for most of the applied application variables.

Table 6 Regression results for the segment of small enterprises

Segment of small enterprises (S)	Estimator ß	Std. error	Z value	Pr(>|z|)	Significance
c1	–2.58767	0.01922	–134.63505	< 2E–16	***
WOEWBH_DNP60_71MP_R	–0.81597	0.01655	–49.29403	< 2E–16	***
WOEWI72	–0.25233	0.03153	–8.00404	1.20403E–15	***
WOEWI1	–0.53027	0.02765	–19.17474	< 2E–16	***
WOEWI27	–0.22311	0.03623	–6.15810	7.36236E–10	***
WOEWI30	–0.48226	0.03254	–14.82144	< 2E–16	***
WOEWI39	–0.33080	0.03000	–11.02689	< 2E–16	***
DMY_GRAD	0.23379	0.04305	5.43094	5.60577E–08	***
signif. codes	‘***’ 0.001	‘**’ 0.01	‘*’ 0.05	‘.’ 0.1	‘ ’ 0
residual deviance:	27475.1	residual degrees of freedom:	64818		
null deviance:	33824.6	null degrees of freedom:	64825		
AIC	27491.1	iterations:	6		

Source: Author’s calculation.

Table 7 Regression results for the segment of medium-sized and large enterprises

Segment of medium-sized and large enterprises (ML)	Estimator ß	Std. error	Z value	Pr(>|z|)	Significance
c1	–2.13712	0.06476	–33.00142	< 2E–16	***
WOEWBH_DNP60_71MP_R	–0.62819	0.05683	–11.05450	< 2E–16	***
WOEWI1	–0.67525	0.10908	–6.19049	5.99758E–10	***
WOEWI30	–0.74384	0.09210	–8.07674	6.65221E–16	***
WOEWI39	–0.57762	0.10751	–5.37280	7.75231E–08	***
DMY_GRAD	0.38332	0.17550	2.18417	0.028949917	*
signif. codes	‘***’ 0.001	‘**’ 0.01	‘*’ 0.05	‘.’ 0.1	‘ ’ 0
residual deviance:	2140.3	residual degrees of freedom:	4217		
null deviance:	2682.3	null degrees of freedom:	4222		
AIC	2152.3	iterations:	6		

Source: Author’s calculation.

Table 8 List of model variables and applied transformations

Original variable name	Variable description	Mark after transformation		Model	
		winsorisation	WoE	S	ML
BH_DNP60_71MP_R	share of average overdue claims up to 60 days 7-1 month before GFI-POD date in total average credit exposure	WBH_DNP60_71MP_R	WOEWBH_DNP60_71MP_R	1	1
I1	current ratio: current assets / short-term liabilities	WI1	WOEWI1	1	1
I27	net liability coverage after-tax profit + amortization / (debt* – reserves) / 365	WI27	WOEWI27	1	0
I30	debt service coverage ratio, DSCR II: EBITD / debt due to FI	WI30	WOEWI30	1	1
I39	payables turnover: accounts payable/ sales revenue	WI39	WOEWI39	1	1
I72	profitability indicator III: income from regular operations / total liabilities	WI72	WOEWI72	1	0
DMY_GRAD	indicator variable: belonging to construciton sector	not transformed	not transformed	1	1

Source: Author’s calculation.

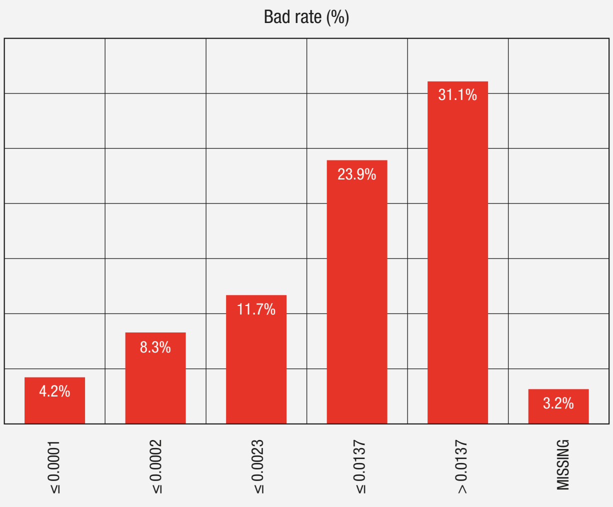
correlation (a lower value of the transformed variable corresponds with a higher level of default risk, so that the expected sign of the estimator β is negative, which corresponds with the signs of regression estimators).

Dummy variables are not transformed, with the result that the signs of their estimators correspond with the signs in the set hypothesis (economic meaningfulness).

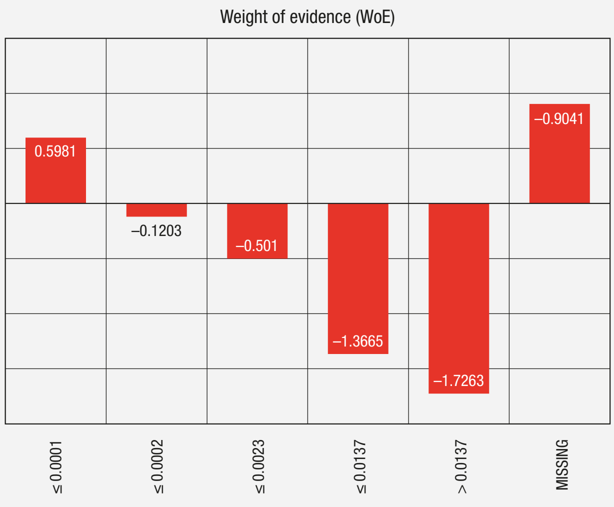
Table 9 Confirmation of the economic meaningfulness of regression parameters: the segment of small enterprises (S) and the segment of medium-sized and large enterprises (ML)

Indicator mark	Economic meaningfulness	Economic meaningfulness OK?	Transfor-mation	Transformation variable risk	Expected sign ß	ß	Signifi-cance	Sign OK?
BH_DNP60_71MP_R	lower is better	OK	WoE	lower is riskier	–	S: –0.8160	0.1%	OK
						ML: –0.6282	0.1%	OK

Default rate on the indicator scale

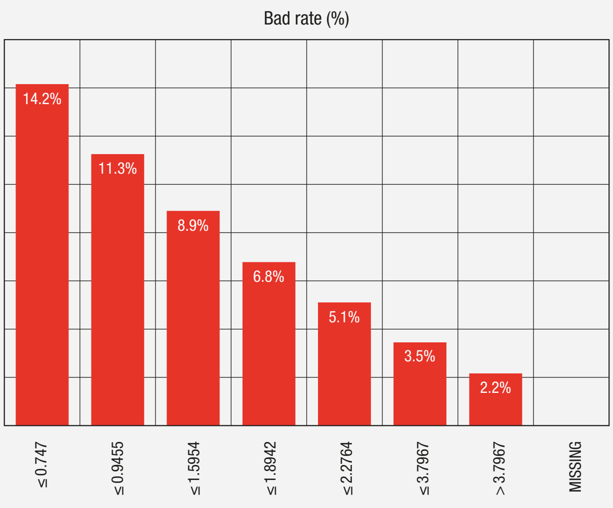


WoE transformation

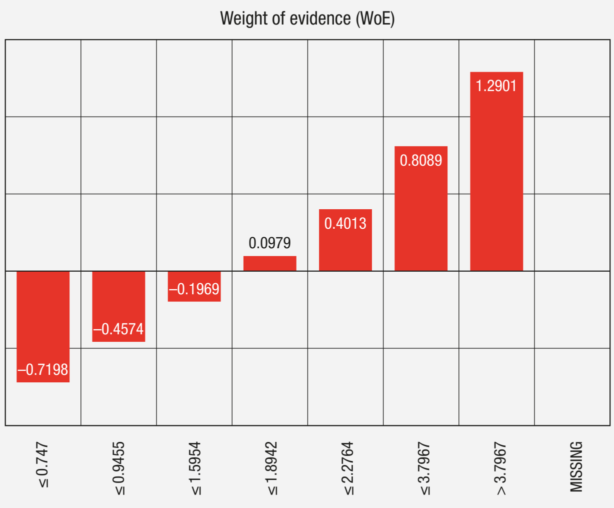


Indicator mark	Economic meaningfulness	Economic meaningfulness OK?	Transfor-mation	Transformation variable risk	Expected sign ß	ß	Signifi-cance	Sign OK?
I72	more is better	OK	WoE	lower is riskier	–	S: –0.2523	0.1%	OK

Default rate on the indicator scale

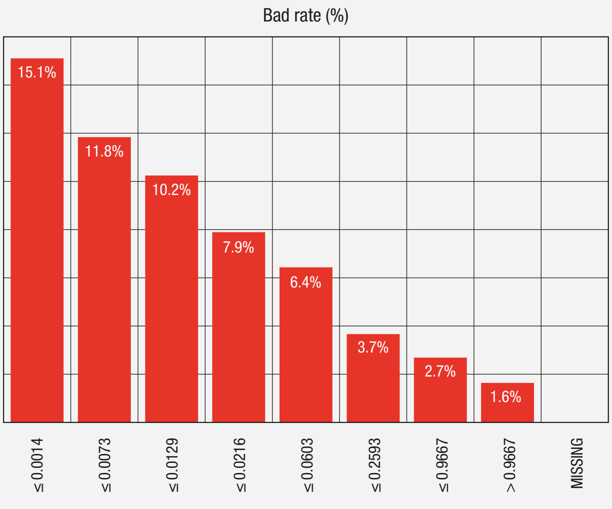


WoE transformation

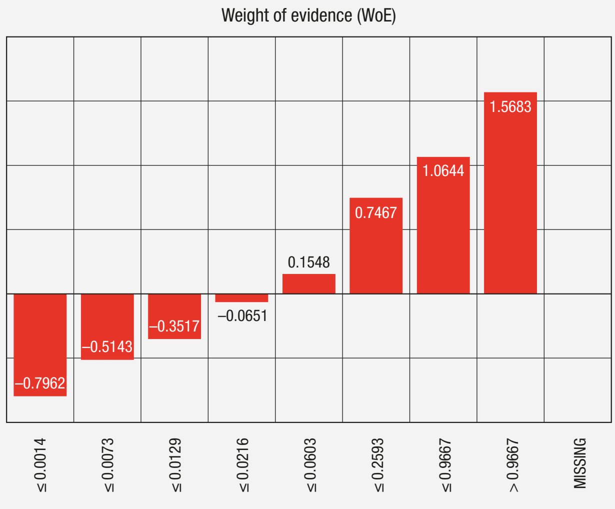


Indicator mark	Economic meaningfulness	Economic meaningfulness OK?	Transfor-mation	Transformation variable risk	Expected sign ß	ß	Signifi-cance	Sign OK?
I1	more is better	OK	WoE	lower is riskier	–	S: –0.5303	0.1%	OK
						ML: –0.6753	0.1%	OK

Default rate on the indicator scale

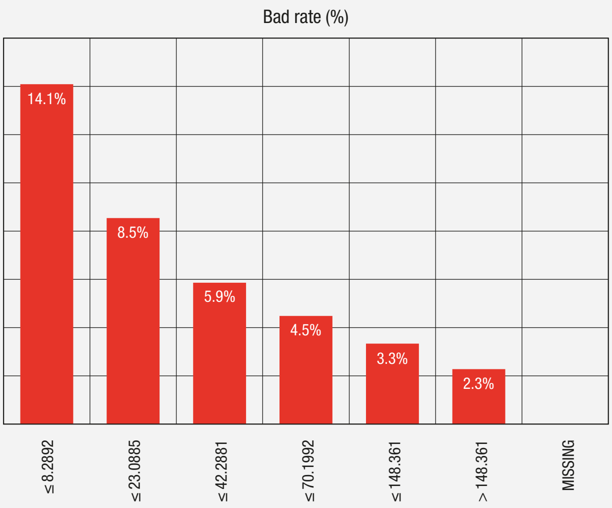


WoE transformation

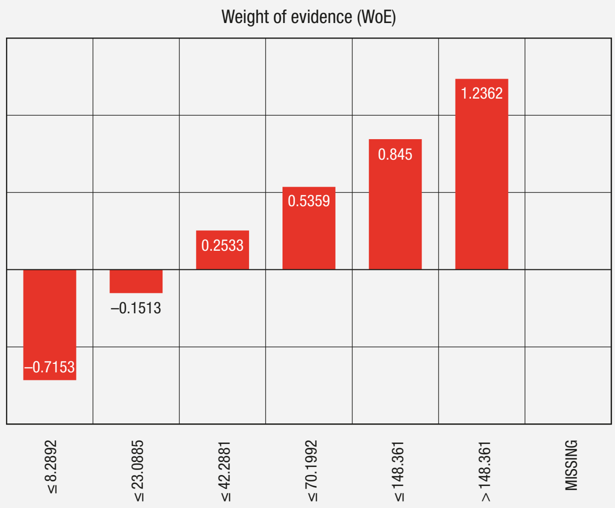


Indicator mark	Economic meaningfulness	Economic meaningfulness OK?	Transfor-mation	Transformation variable risk	Expected sign ß	ß	Signifi-cance	Sign OK?
I27	more is better	OK	WoE	lower is riskier	–	S: –0.2231	0.1%	OK

Default rate on the indicator scale

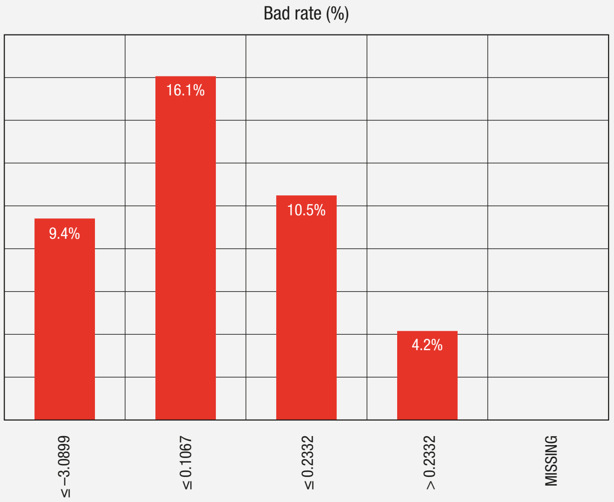


WoE transformation

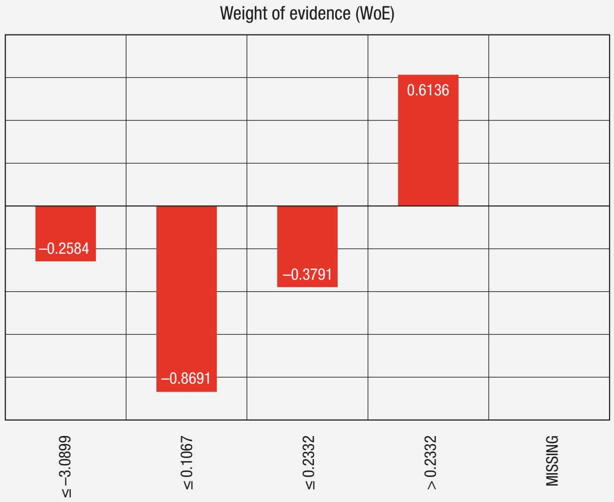


Indicator mark	Economic meaningfulness	Economic meaningfulness OK?	Transfor-mation	Transformation variable risk	Expected sign ß	ß	Signifi-cance	Sign OK?
I30	more is better	OK	WoE	lower is riskier	–	S: –0,4823	0,1%	OK
						ML: –0,7438	0,1%	OK

Default rate on the indicator scale

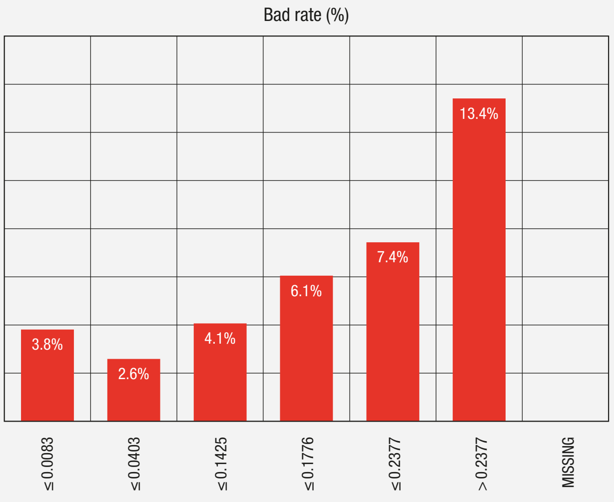


WoE transformation

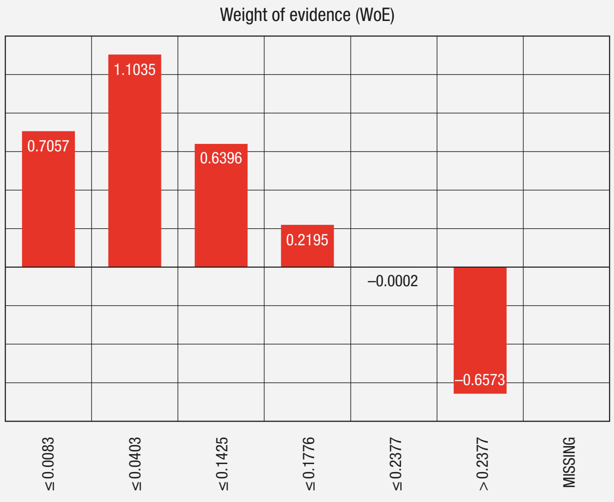


Indicator mark	Economic meaningfulness	Economic meaningfulness OK?	Transfor-mation	Transformation variable risk	Expected sign ß	ß	Signifi-cance	Sign OK?
I39	lower is better	OK	WoE	lower is riskier	–	S: –0.3308	0.1%	OK
						ML: –0.5776	0.1%	OK

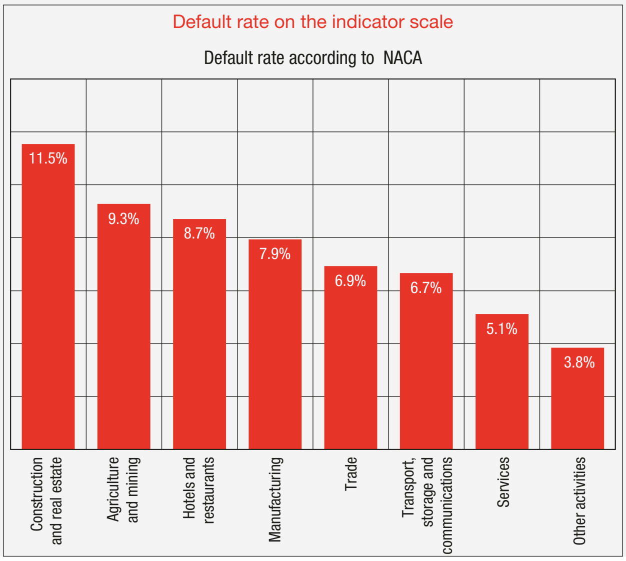
Default rate on the indicator scale



WoE transformation



Indicator mark	Economic meaningfulness	Economic meaningfulness OK?	Transfor-mation	Transformation variable risk	Expected sign ß	ß	Signifi-cance	Sign OK?
DMY_GRAD	less (0) is better	OK	–	more is riskier	+	0.2338	0.1%	OK



Source: Author's calculation.

The behavioural variable *BH_DNP60_71MP_R* has the lowest default rate for the enterprises without this variable value (the value MISSING in the graph), which means that no default on any credit obligation was recorded in the observed period. As expected, as the value of the variable rises so does risk, expressed by the default rate, which is in conformity with the economic meaningfulness of the variable. The resulting sign of the transformed variable estimator is also in tune with economic meaningfulness and expectations (in a WoE transformation, a lower value of the transformation represents a higher risk, which applies to all transformed variables).

The regression results for the financial indicators *I72*, *I1* and *I27* correspond with their economic sense, i.e., a higher indicator value points to a lower enterprise risk, which applies to the whole range of indicators. According to expectations, the signs of regression parameters are negative.

The risk function for indicators 130 and 139 is not monotonic, but inflected in the initial part of the indicator value interval (for both indicators in the second category), which is rectified by the WoE transformation by assigning the weight of evidence to each category that is inversely proportional to the risk it carries.

The dummy variable based on the National Classification of Activities (NACE) emphasises the risk of enterprises from construction and real estate activities, while the positive regression parameter contributes to an increase in the probability of default for enterprises in this activity.

5.3.2 Impact of behavioural variables on the model's discriminatory power

The discriminatory power of the models was tested using the CAP curve and Gini coefficient (see chapter 5.1.2 Initial selection of variables based on univariate analysis). The parameters were also estimated on equivalent samples without behavioural variables, only by means of application and dummy variables. The application models achieved in this way were then applied to calculate the adequate PD for the whole training sample. As expected, models using behavioural variables show better results than models using only application variables.

The application models for both segments have **acceptable** discriminatory power, while the same models extended by behavioural variables have **excellent** discriminatory power (Table 2 and Table 10).

The discriminatory power of the model and the difference between the discriminatory power of the application model and that of the behavioural model can also be observed on the CAP curves according to the areas delineated by the curves of the actual and accidental models (Figure 9 and Figure 10).

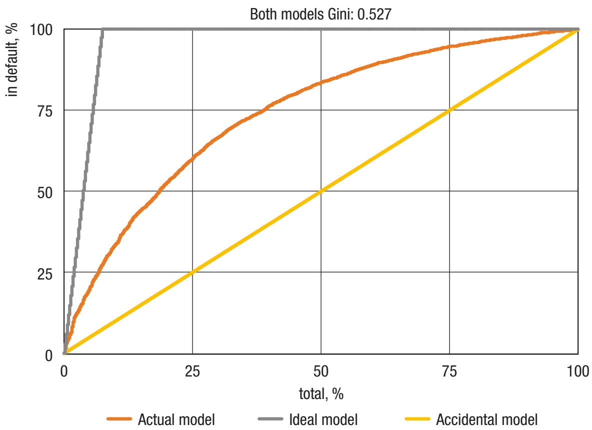
The application of behavioural variables is justified both because they increase the model's discriminatory

Table 10 Resulting Gini coefficients of application and behavioural variables

Model	Gini coefficient (application model)	Gini coefficient (behavioural model)
Medium-sized and large (ML)	0.560	0.615
Small (S)	0.524	0.626
Both models	0.527	0.626

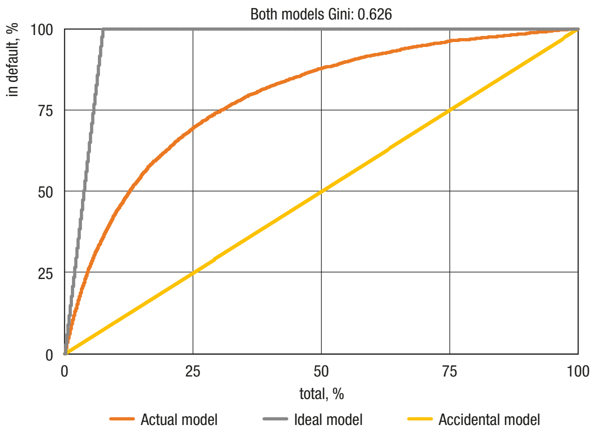
Source: Author's calculation.

Figure 9 CAP curve and Gini coefficient for both segments
application models



Source: Author's calculation.

Figure 10. CAP curve and Gini coefficient for both segments
behavioural models



Source: Author's calculation.

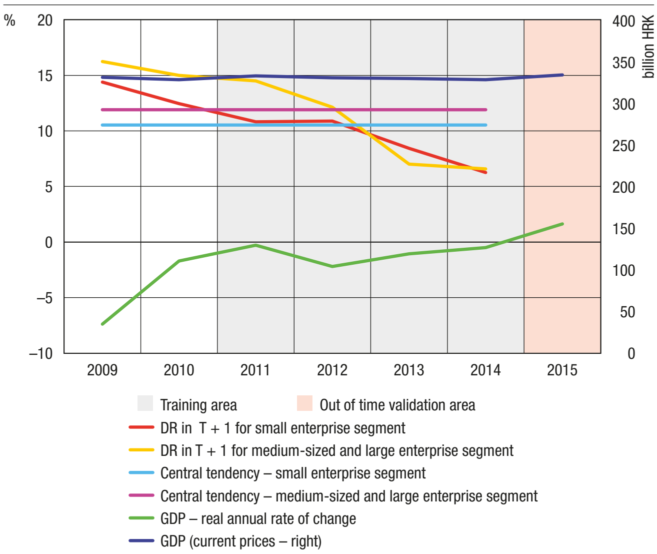
power and because they can be monitored with greater frequency than application variables. The models used behavioural information on obligation payments delayed up to 60 days in the six-month time window (from seven months to one month before the reference date of the annual financial reports of entrepreneurs) for the segments of small enterprises and of medium-sized and large enterprises. Despite the fact that such sporadic delays of up to 60 days do not constitute the initial phase of default, which occurs after a 90-day delay (such cases are excluded from the sample, Figure 1), because the recorded delay of up to 60 days is followed by a longer or shorter recovery period, such enterprises with a high probability of default still encounter more serious problems within a year, that is, they get into default and/or they are reclassified into non-performing clients of credit institutions. Although application variables may not warn of problems in operation, precisely because they are refreshed at a lower frequency (annually), behavioural variables are continuously monitored and credit institutions report to the CNB on such obligors on a monthly basis, so that such information is available throughout the year.

The models use the same application variables (e.g. according to *GFI-POD* for 2015) and refresh behavioural variables with the most recent data (within identical time windows, only with a shift in the reference date) to calculate the new probability of default and rating for such enterprises. This provides for an almost continuous monitoring of the risk of enterprises, enterprise groups or credit institutions' portfolios as well as of the whole system of non-financial corporations.

5.3.3 Calibration of the model and the definition of the rating scale

The model-calculated value $PD_{i\ unkalib.}$ is the uncalibrated value of the probability of default of an enterprise, which reflects the probability of default in relation to the sample on which coefficients β were estimated. The probability of default calculated in this way has to be calibrated according to the central tendency of the default rate of each segment in a given period. This period should, if possible, encompass the whole phase of the economic cycle. However, due to the unavailability of data for older dates, the central tendency was calculated for

Figure 11 GDP trends, default rates and central tendency calculation



Sources: CNB and author's calculation.

Table 11 Default rates and central tendency

Segment	DR^S	DR^{CT}
Medium-sized and large (ML)	9.66%	11.91%
Small (S)	7.28%	10.54%

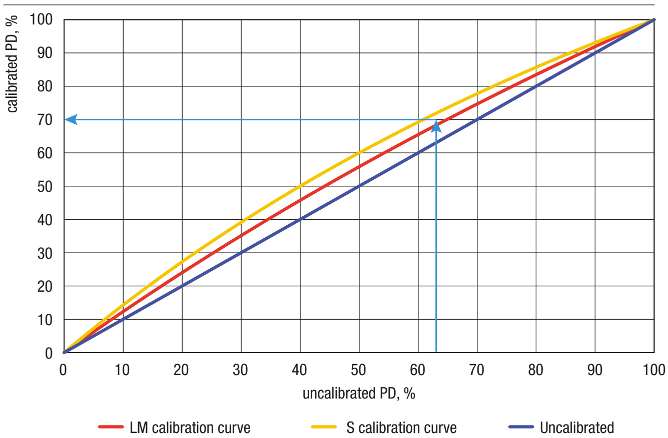
Source: Author's calculation.

default rates in the period between 2010 and 2015, which complies with the PD calculation based on application and behavioural data for the period between 2009 and 2014.

The applied default rates (DR) and central tendencies for the calibration of the model for segments are shown in Table 11.

The definition of the rating scale of enterprise PD enables the monitoring of the "deterioration" of the portfolios of performing clients of credit institutions and spotting potentially problematic enterprises prior to their defaulting. The rating-systems aligned with the IRB approach have to have a rating scale that reflects only

Figure 12 PD calibration according to central tendency



Source: Author's calculation.

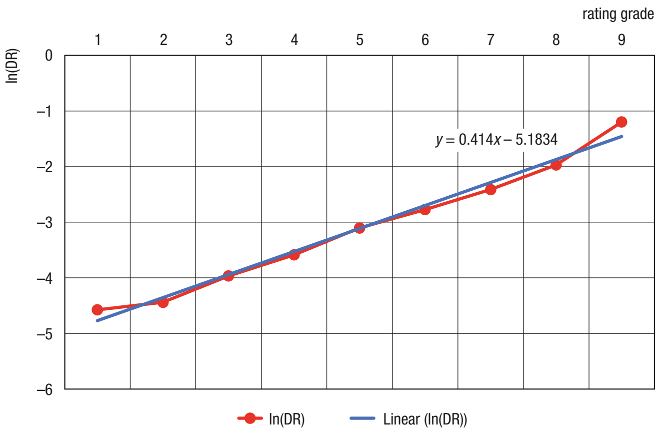
the risk of default and consists of at least seven rating grades for performing clients and at least one grade for non-performing clients[10]. The number of rating grades is usually determined according to the level of diversification targeted by a rating system.

The limits and number of rating grades can also be determined by the supervised discretization, used to calculate WoE transformations and information values. This method exploits the characteristics of such categories (rating grades): the maximisation of differences between categories, that is, rating grades, and the minimisation of the variance within a category.

The creation of rating grades based on annual rates of default contributes to "avoiding" potential shortcomings in the existing PD estimates (or in the risk ranking of obligors in general), because we use data on realised default rates (DR). This method requires an iterative analysis of the linear relationship between the natural logarithm of the default rate $\ln(DR)$ and the rating grades (a logarithmic scale). A rating scale defined in such a way reflects the exponential growth of enterprise risk with an increase in a rating grade, which ensures discrimination between the risk of enterprises in neighbouring rating grades (risk grows almost three times as much as one rating grade):

$$PD \sim e^{rating} \tag{14}$$

Figure 13 Linear relationship between ln (DR) and rating categories – basis



Source: Author's calculation.

Each redefinition of the rating-scale changes the dots for which the line on the logarithm scale needs to be adjusted. The best choice is a line with a minimum measure of adjustment. The beginning of iteration is a line achieved on the rating-scale by clustering with the maximisation of the information value (Figure 13).

The defined rating-scale must meet the following additional conditions:

- Excessive concentration is avoided. A common criterion is that the share of a specific rating grade (by number and volume) should not exceed 25%.
- The rating-scale provides for meaningful ranges of PDs.

This method results in nine rating grades (Table 12, Figure 14, Figure 15). Table 12 shows the distribution of "good" and "bad" enterprises as well as of the total number of enterprises by rating grades, the realised default rate (DR), the average calibrated PD of a rating grade PD, the range and limits of the PD of a rating grade, the condition of a minimum number of observations for a rating grade for the application of the approximation of the binomial distribution by the normal distribution[11] N_{min}, the lower and upper critical value[12]

10 Part 3, Title II, Chapter 3 of Regulation 575.

11 See expression (11).

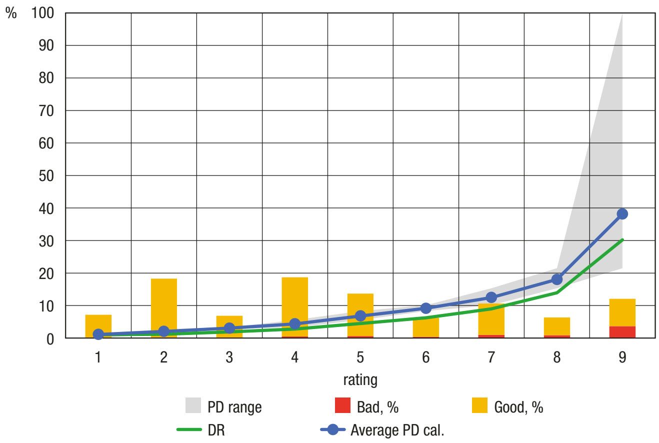
12 See expression (12) and (13).

Table 12 Rating scale design

Rating scale design													CL	0.95			
Distribution													Binomial test				
Grade	Good	Bad	Total	DR	Good, %	Bad, %	% Total	Average PD cal.	Expected DFLT	PD range	PD MIN	PD MAX	Nmin	Inf	Sup	Test Inf	Test Sup
1	4,895	51	4,946	1.03%	7.09%	0.07%	7.16%	1.11%	54.79	1.40%	0.00%	1.40%	822	0.86%	1.35%	ok	ok
2	12,479	149	12,628	1.18%	18.07%	0.22%	18.29%	2.04%	257.31	1.32%	1.40%	2.72%	451	1.83%	2.24%	ne	ok
3	4,658	90	4,748	1.90%	6.75%	0.13%	6.88%	3.05%	144.62	0.62%	2.72%	3.34%	305	2.64%	3.46%	ne	ok
4	12,560	358	12,918	2.77%	18.19%	0.52%	18.71%	4.36%	563.62	2.20%	3.34%	5.54%	216	4.07%	4.66%	ne	ok
5	9,015	424	9,439	4.49%	13.06%	0.61%	13.67%	6.81%	642.70	2.78%	5.54%	8.32%	142	6.38%	7.24%	ne	ok
6	4,045	270	4,315	6.26%	5.86%	0.39%	6.25%	9.15%	394.82	1.79%	8.32%	10.11%	108	8.43%	9.87%	ne	ok
7	6,687	659	7,346	8.97%	9.68%	0.95%	10.64%	12.48%	916.66	5.22%	10.11%	15.33%	82	11.84%	13.11%	ne	ok
8	3,764	610	4,374	13.95%	5.45%	0.88%	6.33%	18.04%	789.25	6.16%	15.33%	21.49%	61	17.09%	19.00%	ne	ok
9	5,817	2518	8,335	30.21%	8.42%	3.65%	12.07%	38.18%	3182.25	78.50%	21.49%	99.99%	38	37.30%	39.05%	ne	ok
Total	63,920	5,129	69,049	7.42806%				10.060%									

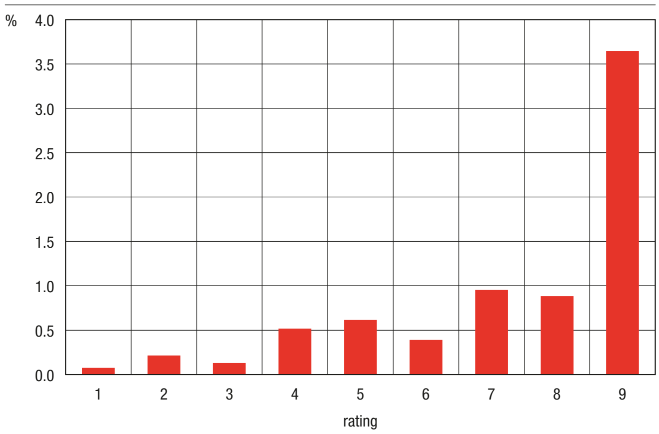
Source: Author's calculation.

Figure 14 Distribution of enterprises, DR and PD by rating grades



Source: Author's calculation.

Figure 15 Distribution of bad enterprises by rating grades



Source: Author's calculation.

Table 13 Rating scale

Description	Rating grade mark	Rating grade numeral	PD_{min}	Average PD	PD_{max}
Enterprises with lowest probability of default	A0	1	0.00%	1.11%	1.40%
Enterprises with very low probability of default	A1	2	1.40%	2.04%	2.72%
Enterprises with low probability of default	A2	3	2.72%	3.05%	3.34%
Enterprises with increased probability of default	A3	4	3.34%	4.36%	5.54%
Enterprises with below-average probability of default	A4	5	5.54%	6.81%	8.32%
Enterprises with average probability of default	A5	6	8.32%	9.15%	10.11%
Enterprises with above-average probability of default	A6	7	10.11%	12.48%	15.33%
Enterprises with high probability of default	A7	8	15.33%	18.04%	21.49%
Enterprises with very high probability of default	A8	9	21.49%	38.18%	99.99%
Enterprises in default, classified in A90	A9	10			
Enterprises in default, classified in B1	B1	11			
Enterprises in default, classified in B2	B2	12		100%	
Enterprises in default, classified in B3	B3	13			
Enterprises in default, classified in CC	CC	14			

Source: Author's calculation.

with a reliability limit of $\alpha = 95\%$ and the results of the binomial test at the lower and upper limit of reliability. The green fields signify the fulfilment of the assumptions of the binomial test and the red fields signify that the assumptions were not fulfilled.

Binomial tests reflect a margin of conservatism in all rating grades, which was achieved by the calibration of PD according to the central tendency and comprises most of the recession phase of the cycle and only the initial phase of exit from recession. The number of performing rating grades meets the IRB conditions for the rating scale, prescribing at least seven grades for performing exposures and one grade for non-performing exposures, that is, enterprises. The concentrations of the number of enterprises are higher in the first two rating grades, but they still remain below the usual limit of 25% for excessive concentrations. Such increased concentration in the first two rating grades suggests that the differences among enterprises in high grades are very small and hard to measure, even when behavioural variables are applied. However, behavioural variables improve the classification of enterprises according to their risk precisely in the part of the scale with poorer rating grades. This research also included the testing of a rating scale with a finer distribution at its beginning. However, precisely due to the lower discriminatory power of application variables that prevail in the calculation of PD of the best enterprises and a small number of enterprises in the best rating grade, the results of binomial tests over time are not completely satisfactory so that the first version of the scale was retained, which is satisfactory in view of the requirements of risk assessment for the portfolio of non-financial corporations. Credit institutions will in this part have a finer distribution of the rating scale due to the need to differentiate more precisely between the quality of obligors in the loan underwriting process, which is not the case here. The final rating scale is shown in Table 13.

The table consists of nine rating grades for performing enterprises, while for non-performing enterprises the existing rating grades were retained, according to "A90 days"[13], "B1", "B2", "B3" categories for party recoverable placements and "CC" for fully irrecoverable placements. The final rating scale thus has 9 + 5 rating grades.

13 The Decision on the classification of exposures into risk categories and the method of determining credit losses (OG 114/2017) revoked A90 risk category.

6 Conclusion

Obeying the rules of the development of the PD model as defined by regulations and the best business practice, two PD models were developed for the sector of non-financial corporations: one for the segment of small enterprises and the other for the segment of medium-sized and large enterprises. The choice of segmentation was primarily determined by the differences between risk characteristics of the segments thus selected, while risk factors included in the final models reflected specificities, but also similarities in the operation of the segments. Irrespective of the segment, in the analysed period construction enterprises were riskier than enterprises from other industrial activities. The increased risk of both segments was also influenced by the unfavourable relations of liquidity indicators, payables turnover and debt service coverage ratio. Although the indicators of profitability and coverage of net obligations are risk factors that have a significant impact on the probability of default in the segment of small enterprises, they do not considerably influence risk increase in the segment of medium-sized and large enterprises due to the different business models, which leave medium-sized and large enterprises more room and greater possibilities for additional asset management, making them less sensitive to volatility of operating income and realised profit. The models also included one behavioural variable, used to detect early warning signs of difficulties in settling obligations due to credit institutions, which spontaneously and independently improve before the observation period. The introduction of the behavioural variable into the models considerably improved their discriminatory power relative to the models with the same risk factors, but without a behavioural variable. PD estimates were calibrated and mapped into a new rating scale, and the results of the validation tests made on the training sample and out of time sample are very good and good, respectively[14].

The development of the PD model in accordance with the Basel III regulation and the application of the principle of the best business practice resulted in a PD model that fulfils the most important criteria from the areas of discriminatory power, classification precision, calibration and stability and is a valuable instrument in the measuring of the component of credit risk exposure of the financial system to the sector of non-financial enterprises.

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14 The results of validation tests are available on request.

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Glossary and abbreviations

application data – data that are usually collected from the client of a credit institution on the receipt of the loan application; data on enterprises include those collected from annual financial reports and credit worthiness information (*BON-1, BON-2*), qualitative data from the application form, etc.

AR – accuracy ratio: measure of the model's discriminatory power derived from the CAP curve, also termed the Gini coefficient.

AUROC – Area Under Receiver Operating Characteristic; the area under the ROC curve.

behavioural data – data arising from the business relationship between the client and the credit institution, such as the regularity of repayment of obligations due, the number and type of financial products used by the client and the conditions for their use, promptness in submitting the required information to the credit institution, etc.

CAP curve – Cumulative Accuracy Profile; also known as the Lorenz curve, shows the discriminatory power of the model.

DR – default rate.

EL – expected loss.

GFI-POD – annual financial report of the entrepreneur, compiled based on the forms of the Financial Agency.

dummy variable – variable that takes the value 1 if it indicates the presence of a specified characteristic of an enterprise and the value 0 if the enterprise does not meet the set criterion; used for the assessment of the specified enterprise characteristic (e.g. its belonging to an activity) to its PD estimate.

IRB – Internal Ratings Based Approach; approach based on internal ratings according to Basel II/III regulations, prescribed by Regulation (EU) No 575/2013 of the European Parliament and of the Council of 26 June 2013 on prudential requirements for credit institutions and investment firms and amending Regulation (EU) No 648/2012.

IV – Information Value; measure of the information value, described in more detail in section 4.2 Weight of evidence (WoE) and information value (IV)

cohort approach – method which shows the change of the initial rating relative to the rating given at the end of the observed period (one year).

confusion matrix (error matrix) – matrix model errors used to calculate the preciseness the model's classification.

duration or **hazard rate approach** – takes into account all changes in a rating in the observed period and the duration of these changes.

OOS – Out-of-Sample; set of data on the enterprises that were not included in the sample that was used for the estimate of the model parameters for the period used in the estimate.

OOT – Out-of-Time; set of data on the enterprises that were included in the sample that was used for the estimate of the model parameters, but outside the period used in the estimate.

observation period – for the assessment of one-year probability of default (PD) the observation period is a period of one year during which the change in default is observed; the initial state is the state on 31 December of the year T_0, and the final outcome at the end of the observation period in the year T_{+1} depends on the change in default in the period from 1 January to 31 December in the year T_{+1}.

PD – probability of default.

overfitting – estimate of the parameters of the regression model that very well describes the phenomenon on the data on which the estimate was made, but the response and characteristics of the model are significantly weakened on other out of sample and/or time data.

ROC curve – receiver operating characteristic curve; curve showing the response as the error function of the classifier.

RWA – risk weighted assets.

STA – Standardised Approach; standardised approach pursuant to Basel II/III regulations, prescribed by Regulation (EU) No 575/2013 of the European Parliament and of the Council of 26 June 2013 on prudential requirements for credit institutions and investment firms and amending Regulation (EU) No 648/2012.

default – defined in Regulation (EU) 575/2013.

training sample – sample on which regression parameters are estimated.

validation sample – sample on which the performances of the regression model are verified (OOT or OOS sample).

WoE – Weight of Evidence; transformation of variables by the weight of evidence is described in more detail in section 4.2 Weight of evidence (WoE) and information value (IV) .

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Additional information, such as acknowledgments, should be incorporate in the text at the end of the introductory section.

The second page should contain the abstract and the key words. The abstract is required to be explicit, descriptive, written in third person, consisting of not more than 250 words (maximum 1500 characters). The abstract should be followed by maximum 5 key words.

A single line spacing and A4 paper size should be used. The text must not be formatted, apart from applying bold and italic script to certain parts of the text. Titles must be numerated and separated from the text by double-line spacing, without formatting.

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