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If Fail, Fail Less: Banks' Decision on Systematic Vs Idiosyncratic Risk

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If fail, fail less: Banks' decision on systematic vs idiosyncratic risk

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Abstract

This paper investigates the influence of bailout policies on the banks' choice between systematic and idiosyncratic risk. The regulator that cares about bailing out banks with higher asset values is

introduced in the 'too many to fail' paradaigm. This creates a regulatory channel that may incentivise banks to differentiate and undertake less systematic risk. The novel feature introduced is the heterogeneity of banks in failure, which creates a richer set of possibilities for the regulator.

I find that once the bank's probability of a bailout depends on the bank's value in failure, and not only on how systemic the banking crisis is, the banks rather choose to invest in the idiosyncratic investments than in the common, market project. Therefore, the herding pressure of the 'too many to fail' guarantees is reduced, as well as the occurence of the systemic banking crises.

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1 Introduction

The banks' behaviour under the existance of the bailout guarantees has been extensively studied in the banking literature. The idea that existance of the lender of last resort distorts the risk-taking incentives of banks goes back to Bagehot [1873]. He was first to dicuss the moral hazard arising from the belief that a troubled bank will be rescued by the regulator.

Over time, the views on the regulator's involvement in the process of bank failure resolutions have evolved, as it has become clear that not all episodes of bank failures are the same. One of the important distinctions that determined the differences in regulatory actions was how systemic and wide-spread bank crises were.

It has been empirically identified that the regulatory interventions seem to be very much dependent on whether the bank failure arises due to some idiosyncratic reasons or due to deterioration of the aggregate conditions. Aggregate conditions may induce simultaneous bank failures which then cause systemic banking crisis. This puts certain constraints on the regulator's set of available resolution policies he can employ in resolving multiple bank failures. Empirical results point out that the regulatory intervention differs in systemic banking crisis versus dealing with individual bank failure, see, e.g. Kasa and Spiegel [1999] and Santomero and Hoffman [1998]. Therefore, depending on how widespread the crisis is, regulators would have different sets of available policies, and perhaps even different criteria for resolving bank failures.

Kasa and Spiegel (1999) provide an empirical analysis of two potential bank closure policies that the regulator can implement. They compare an absolute closure rule according to which the bank is closed when its asset-liability ratio falls below a given threshold, and the relative closure rule in which banks are closed once they are doing worse than the industry average. They provide a convincing evidence that relative closure rule is implemented more often, also finding that it results in lower costs for taxpayers. Similar empirical results have been provided by Brown and Dinç [2011] who show that bailout interventions are more likely if the crisis is systemic, while bank closures happen more often in case of isolated bank failures.

This phenomenon, that banks are more likely to be offered help from the regulator when failing with other banks, rather than on their own, has been termed as

'too many to fail' in the existing literature. It is supposed to capture the feature that bailouts are overall more likely the more systemic is the cirsis. Therefore, the banks will try to choose more correlated investments in order to increase the probability of failing together and thus benefit from the bailout subsidies.

In my model, I analyze the concept of 'too many to fail' guarantees with an additional feature: banks having different values in failure. Even if the overall probability of bailout increases the more banks fail together, each bank's individual bailout probability will depend on its own value, as well. This is what corresponds to the empirical finding that bank's relative performance is what matters for the regulator's bailout decision. Although interventions are more probable when crisis is systemic, the bank's probability of being in the group of the bailed out ones increases if the bank failed less than its failed peers.

Namely, introducing the regulator who cares to preserve better banks in the system should create a connection between the bailout probability of the bank and its preformance in failure. Then, I try to analyse how this feature of the bailout policy affects banks' investment behaviour, particularly focusing on the banks' choice between systematic and idiosyncratic risk. If it is only important that many banks fail together, banks would choose to undertake as much correlated, systemic risk, in order to ensure that they would fail with others. However, if bank's individual value in failure also matters for the bailout probability, this would reduce the herding pressure, as banks would try to have higher values in failure, to ensure the bailout.

Models focusing on 'too many to fail' guarantees analyse the effect of these guarantees on bank's choice of interbank correlation (Acharya and Yorulmazer [2007, 2008]).What Acharya and Yorulmazer find is that 'too many to fail' guarantees induce banks to choose correlated investments in order to increase the probability of failing together. If a bank fails on its own, the regulator has other available resolution options, such as selling the bank to the surviving banks or even outside investors, which makes bailouts less probable (Acharya and Yorulmazer [2008]). Therefore, banks prefer to undertake correlated investment by lending to similar industries, betting on common risks, issuing syndicated loans and the like, in order to be able to capture the bailout subsidies which exist only if they fail together.

Thus, 'too many to fail' guarantees represent a channel explaining the herding behaviour of banks. In addition, there are other potential channels that

may explain banks' herding behaviour. Rajan [1994] elaborates on reputational considerations of bank-managers. He investigates how short horizons combined with reputational concerns may result in bank managers manipulating current earnings in order to hide bad loans, and reveal them only when the entire borrowing sector is hit by an adverse systematic shock. This creates coordination in banks' credit policies, therefore resulting in banks' herding behaviour. Penati and Protopapadakis [1988] and Mitchell [1998] explain bank herding with a greater focus on the regulatory dimension. Namely, increased probability of bailout interventions when banks fail together may induce the banks to undertake inefficient investments in the common markets, or again coordinate when disclosing losses, all resulting in herding behaviour of banks.

However, one should not neglect the existence of the opposing forces that induce strategic substitutability among banks' actions. For instance, Perotti and Suarez [2002] describe a model in which bank prefers to be the surviving one, as it gains in higher charter value once the market becomes more concentrated due to bank failures. Acharya and Yorulmazer [2008] on top of elaborating on the 'too many to fail' herding channel, explain that ability to purchase the failing banks' assets at a discount, due to fire-sales or particular liquidity provision policy implemented by the regulator, may create incentives for banks to pick lower interbank correlation in order to be the surviving ones.

In my model, another channel which should provide incentives for banks to differentiate and choose less correlated investment is introduced. After the crisis has become systemic enough for the regulator to intervene, he cares to bailout the banks that failed less relative to their peers.

Banks are the same ex ante, in terms of investment opportunities they are endowed with. However, I introduce the heterogeneity in failure, such that once project cash flows are realized, the banks may have different values in failure, given the investment decision they have initially made. This implies that the regulator might not be indifferent regarding which banks to liquidate and which ones to bailout. Given that the regulator cares about bank's value when deciding who to bailout, the bailout probability will depend on the aggregate state, determined by the total number of failures, but also on the bank's individual portfolio choice and its preformance in failure.

The feature that the regulator prefers to bailout the better banks is endogenized by defining each bank's future asset value, i.e. future cash flow from the project,

as a function of the date 1 cash flow realization. Then, a value maximizing regulator wishes to preserve ex post better banks in the system, banks that failed less. In this setup, it is reasonable to expect that this changes bank's investment decision from the perspective of systematic risk it undertakes, relative to the case in which bank only cares about failing with others. Therefore, this type of bailout policy may potentially reduce the banks' willingness to herd ex ante.

Furthermore, in this setup I am able to investigate how the anticipated heterogeneity in failure affects the portfolio choice once banks are heterogeneous ex ante. I introduce different bank types, endowed with different bank-specific investment opportunities, in order to explore how the portfolio decision varies across bank types.

Starting with Bagehot [1873] there is extant literature focusing on the effects of implied bailout guarantees on the bank's risk-taking incentives and the issue of moral hazard. In that respect, Mailath and Mester [1994] study the time-inconsistency of different bailout policies from the perspective of bank failures being costly. According to Mailath and Mester (1994) bank closures cause disruption of the payment system, hurt investor confidence in other banks and dissipate information capital on borrowers. In order to avoid these costs, bailouts happen more often than it is optimal ex ante. Similarly, Huang and Goodhart [1999] further explore regulator's role as the lender of last resort, by assuming that there are costs related to bank closures which not even the regulator can perfectly observe. Therefore, bailouts might still represent a preferred resolution in spite of the moral hazard they induce. In that respect, Freixas [2000] provides a cost benefit analysis of bailouts, deriving the constructive ambiguity as a potential way to overcome the negative effects induced by the bailout guarantees. All of the papers mentioned focus on the single-bank model, abstracting from the potential strategic interactions among banks and the implications of the regulator's policy on these interactions.

Among the models that introduce state-contingency in the regulator's bailout policy and thus try to capture the feature that interventions are more likely when the market is overall doing poorly is the paper by Nagarajan and Sealey [1995]. They introduce the state-contingent acting regulator who implements the forebearance policy depending on the overall state of the economy. Analysis of bank's moral hazard problem under forebearance policy shows that when failing banks are not closed if the market overall does poorly, this can alleviate bank's moral hazard problem ex ante. Similarly, Cordella and Yeyati [2003]

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show that if the regulator is able to commit ex ante to a particular bailout policy in which he intervenes once the crisis is systemic, this again mitigates bank's moral hazard problem. The banks choose to be more prudent and cautious with investment, due to the value effect arising from the increase in banks' charter values. In Nagarajan and Sealey (1995) the banks are not allowed to choose their degree of correlation with the market, but can only choose the type of the idiosyncratic project. Therefore, the analysis of the effect of state-contingent regulator on strategic interaction among banks is still missing. In Cordella and Yeyati (2003) the crisis becomes systemic due to exogenous adverse macroeconomic shocks, thus the effect of banks' choices on the creation of systemic crisis is neglected. Acharya and Yorulmazer [2007] address this issue by endogenizing the systemic nature of the crisis, since the crisis becomes systemic as a result of banks' overinvestment in perfectly correlated industries.

In my model, banks are also able to influence the extent of the crisis by their investment decisions, and consequently, regulator's policy will depend on the total number of banks failing. However, unlike Acharya and Yorulmazer [2007], the investment projects have different cash flow realizations in the low state which allows me to have the banks that are heterogeneous ex post, depending on their investment decision. This describes a more realistic setup and introduces a broader spectrum of possibilities for the regulator's bailout policy.

In the paper, I first introduce the benchmark model in which the banks will be the same in failure, regardless of their investment decision. This setup analyzes the effect of the 'too many to fail' bailout policy on the banks' ex ante choice of systematic vs idiosyncratic risk. Namely, the more banks fail together, the greater probability of a bailout for each failed bank. The bailout here is only contingent on the aggregate state, defined by the total number of banks that fail. Since banks prefer to capture the bailout subsidies, they choose to herd and put greater weights on the systematic investment i.e. the market project.

Second, I change the setup in order to introduce the regulator that prefers bailing out only the ex post better banks i.e. the ones that failed less relative to other failing banks. Here, the individual bank's bailout probability depends on the total amount of failures i.e. how systemic the crisis is, but also on that bank's value in failure, since this value will determine bank's continuation value as well. This is what introduces the incentive for banks to differentiate ex ante and undertake less of a systematic risk. Therefore, banks still care to fail together, however they wish to 'fail less' relative to others which induces the

change in the investment decision relative to the 'too many to fail' benchmark.

The 'fail less' bailout policy in which banks that failed less are favoured by the regulator, induces undertaking less of the systematic risk ex ante. Hence, the banks choose to invest in the bank-specific project and undertake more idiosyncratic risk, once their value affects their chances of being bailed out and capturing the cash flows from the next period.

In order to analyse welfare implications of the 'fail less' bailout policy in a more general setup, I introduce the ex ante heterogeneity among banks through the different returns of the bank-specific projects. It turns out that the existance of banks endowed with poor bank-specific projects, i.e. bad banks, actually strenghtens the no herding incentives for good banks. From the welfare perspective, 'fail less' bailout policy always dominates the 'too many to fail' policy, since bailing out banks with higher values in failure leads to banks choosing more efficient investment strategies that result in higher welfare. Also, the 'fail less' bailout policy is time consistent, since commiting ex ante to bail out banks with higher values, if failures occur, is also optimal to be done ex post.

The remainder of the paper is structured as follows. Section 2 presents the setup of the model. Section 3 analyzes 'too many to fail' benchmark model, while Section 4 introduces the heterogeneity in failure and the 'fail less' model. Section 5 discusses the welfare implications of the 'fail less' bailout policy when banks are identical in the first period. Section 6 introduces banks that are heterogeneous in the first period and discusses their optimal investment strategies under 'fail less' bailout policy and welfare implications. Finally, Section 7 concludes. The proofs and derivations that are not in the text are given in the Appendix.

2 Model setup

Players

In this economy, there are three dates $t = 0, 1, 2$, n risk-neutral banks, and one risk-neutral Regulator. Each Bank i can borrow from a continuum of depositors of measure 1. At date 0 depositors deposit their one unit of endowement in the bank. Deposits are in the form of standard debt contract such that depositors are promised to receive return r at date 1. Deposits are assumed to be fully insured. In addition, there also exist the outside investors endowed with

some liquidty to be spent on purchasing the banking assets when offered by the Regulator.

Banks

Each bank has access to the common investment project $\tilde{R}_m$, which I refer to as the market investment project. In addition, each bank is endowed with the bank-specific, investment project $\tilde{R}_i$. Bank-specific nvestment projects' are independent among each other, and independent from the market. Given that all banks can access the market project and so achieve perfect correlation of investment portfolios, market project represents the systematic, correlated risk. Bank-specific projects, being independent, represent the idiosyncratic risk banks can undertake.

Both market and bank-specific projects are two-period projects. First-period cash flows are high $\overline{R}_{1j}$ with probability α, or low $\underline{R}_{1j}$ with probability $1 - \alpha$, where $j = i, m$. If high cash flows realize the bank is able to repay r to its depositors, regardless of the project in which it invested. Namely, $\overline{R}_m \geqslant \overline{R}_i > r$ is assumed. If low project cash flows realize, since $r > \underline{R}_i \geq \underline{R}_m$ holds, the bank is not able to cover r and is considered failed.

Second-period project cash flows depend on the realization of the first-period cash flows in the following way: In case high cash flows realized at date 1, the cash flows in the second period are equal to V. However, if the bank failed at date 1 cash flows at date 2 will be some function of the cash flow realization at $t = 1$, defined as

$$R_{2j} \mid \underline{R}_{1j} \sim U\left(\underline{R}_{1j} - \varepsilon, \underline{R}_{1j} + \varepsilon\right) \tag{1}$$

Given the bank was able to repay depositors at date 1 or was given the adequate help from the Regulator through the bailout intervention, it will be able to capture the second-period project cash flows.

At date 0, Bank i chooses whether it will invest its depositors' endowment in the idiosyncratic or the market project. I will model this investment decision as $x_i = 1$, if the bank has chosen to invest in the bank-specific project, and $x_i = 0$ if the bank has chosen to invest in the market project. The bank is trying to maximize the total expected value of project cash flows.

Outside Investors

Outside investors are buying failing banks' assets offered to them by the Regulator. However, outside investors are less efficient users of the banking assets. They can produce only $L(k)$ from each bank bought, where k is the number of banks they end up buying .

Consequently, $L(k)$ represents the liquidation value of a failing bank, given the Regulator has decided to sell k banks to the outsiders. Therefore, a liquidation value of the bank is some function of the total number of banks that will be liquidated. Also, liquidation value is decreasing in the number of banks liquidated i.e. $L'(k) < 0$. The more banks are sold to outsiders, the lower the liquidation value of each bank. This assumption relies on the notion of asset-specificity, which was first introduced by Williamson [1988] and Shleifer and Vishny [1992]. They suggest that firms with specific assets typically suffer from 'fire-sale' discounts and lower liquidation values, when sold to investors that are outside of the firms' industry. In case of banks, this phenomenon was empirically studied by James [1991] who showed that a liquidation value of a bank is typically lower than its market value as an ongoing concern.

Decreasing liquidation value of banks represents a reduced form for capturing the increasing social loss the more banks get sold to outsiders, and therefore no longer perform the banking functions.

Outsiders are rational, and the price they are willing to pay for the banking assets is less or equal to the value they are able to produce. In this model, outside investors' always break even and their participation constraint binds:

$$p(k) = L(k) \qquad (2)$$

where $p(k)$ is the price they pay per bank bought, given that in total k banks are being liquidated through sales to outsiders. From here it is obvious that $p(k)$ is also decreasing in k.

Regulator

At date 1, Regulator observes bank failures and decides between bailing out the bank or selling it to outsiders . His objective is to maximize the total

expected output of the banking sector in the following period net of any bailout or liquidation costs.

Costs of the intervention are considered to be the fiscal costs related to covering deposit insurance. I assume the linear fiscal cost function defined as $C(d) = d$, where d represents the number of units of funds needed in order to cover deposit insurance. Fiscal costs of providing funds for deposit insurance can be explained by the standard distortionary effect that increase in taxes, which usually occurs at those times, may have. Also, other potential costs related to banking crises and bank failures that result from immediacy of providing the funds may be captured through this cost function.[1]

Bank has failed if the cash flows generated by its project at date 1 are not sufficient to cover the promised return r to depositors. Since deposits are insured, the Regulator will have to cover the shortfall. Also, the Regulator decides whether to allow the bank to continue operating and thus capture the second-period project cash flows, or to liquidate the bank and its project by selling it to the outside investors, in which case the bank gets nothing.

Given that the outsiders are less efficient users of the banking assets, bank liquidation means that there will be some social loss in terms of unrealized second period project cash flows. However, since outsiders are endowed with some liquidity, and are paying a fair price for the assets they buy, this aditional liquidity Regulator collects would reduce the cost of deposit insurance.

Thus, the Regulator is facing a trade-off between lowering the cost of deposit insurance through bank sales and incurring the social loss from selling the banking assets to less efficient outside users.

In this model, a bailout is considered as a permission to the bank owner to continue operating the bank, regardless of not being able to repay depositors at date 1. On the other hand, once the bank is sold to outside investors, the bank owner loses the right to run the bank, therefore loses entire future cash flows of the bank.

Since deposits are covered regardless of the resolution action, what determines the choice between bailing out the bank or selling it to outside investors is the following:

1) outside investors are not efficient users of the banking assets - loss from sales

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[1]See Calomiris [1998] for a detailed discussion on fiscal costs associated with banking failures and bailouts

2) the amount outside investors pay when purchasing the banks' assets reduces the cost of deposit insurance - gains from sales

The Regulator is able to compare the payoffs of his potential actions for each failed bank. If he bails out the k -th Bank:

$$E\left(R_{2j} \mid \underline{R}_{1j}\right) - \left(r - \underline{R}_{1j}\right)$$

while if he liquidates this k-th Bank through sales to outsiders, the payoff will be

$$L\left(k\right) - \left(r - \underline{R}_{1j} - p\left(k\right)\right)$$

given k banks in total are liquidated. By comparing these payoffs we can see that the net social loss from selling the k-th Bank is $E\left(R_{2j} \mid \underline{R}_{1j}\right) - L\left(k\right)$, while the netgain from selling the k-th Bank is given by the price outsiders pay, when k banks sold in total, $p\left(k\right)$.

Regulator's optimal bailout policy will be defined from his maximization of the total banking output net of all the costs related to his chosen intervention, which implies that it would be optimal for the Regulator to sell the failed banks as long as the gain from selling is higher than the social loss induced by inefficient sales of the banking assets to outsiders.

Let f be the number of failed banks at date 1. The Regulator decides on his bailout policy by trading off the gains of liquidation with the losses until he is indifferent between liquidating or bailing out the marginal bank. Given f banks fail at date 1, there will always be some k^*, where $k^* \leq f$, such that k^* banks are liquidated, while $f - k^*$ banks are bailed out. Then, the k^*-th Bank is the marginal bank for which the Regulator is indifferent between selling or bailing it out, given that the social loss and gains from this liquidation are equated.

For the marginal failed bank k^* the following expression holds:

$$E\left(R_{2j} \mid \underline{R}_{1j}\right) - L\left(k^*\right) = p\left(k^*\right) \tag{3}$$

As long as:

$$E\left(R_{2j} \mid \underline{R}_{1j}\right) - L\left(k\right) < p\left(k\right)$$

it will be optimal to sell the k-th Bank to outsiders.

Lemma 1 *Let f be the number of bank failures that occurs at date 1. Regulator's optimal bailout policy at date 1 will be defined as follows:*

There will be some marginal $k^* - th$ bank, where $k^* \leq f$ such that the following holds:

$$E\left(CF_2\right) - L\left(k^*\right) = p\left(k^*\right)$$

Then, it will be optimal to liquidate k^* banks, while bailout $f - k^*$ of the faling banks, given expressions in (4) hold:

$$\begin{cases} E\left(R_{2j} \mid \underline{R}_{1j}\right) - L\left(k^* - 1\right) < p\left(k^* - 1\right) & and \\ E\left(R_{2j} \mid \underline{R}_{1j}\right) - L\left(k^* + 1\right) > p\left(k^* + 1\right) \end{cases} \tag{4}$$

Timeline

At date 0 depositors deposit their endowements in the banks and are promised return r at date 1. At date 0 each Bank i chooses to invest this one unit of depositors' endowment either in the market project or in the bank-specific investment project. At date1 first-period cash flows of the projects are realized. If the bank is not able to meet its liabilities towards depositors, the bank is considered failed. The Regulator has to choose between liquidating the bank through sales to outside investors or bailing it out.

If the bank is bailed out, the bank owners are allowed to continue running the bank and they capture the second-period cash flows of their projects. If the bank is sold to outsiders, initial bank owners get nothing and outside investors are able to realize only a liquidation value of banks.

3   'Too many to fail' benchmark

In order to analyse the effects of bailout policy on banks' risk-taking and investment decision, I start with a benchmark model that is supposed to capture the 'too many to fail' feature of the bailout policy.

In this setup, I assume that $\bar{R}_m = \bar{R}_i = \bar{R}$ and $\underline{R}_m = \underline{R}_i = \underline{R}$ holds. Namely, the market and bank-specific projects have identical cash flows. By investing in the market project common to all the banks, banks can achieve perfect correlation

of their project cahs flows, while if investing in the bank-specific projects the correlation will be zero.

In case of failure, the Regulator will be indifferent which banks to liquidate and which banks to bailout, once the total number of bailouts and liquidations is determined.

In this setup, banks are homogeneous in failure and face the same bailout probability which only depends on the aggregate statedefined by the total nnumber of failures. Therefore, later I will use this setup as a benchmark for comparison with banks' choices of risk under different bailout policies.

3.1 Solving the Model

Regulator's decision

The banks fail if the low project cash flows have realized at date 1. Since both the market and bank-specific projects have the same cash flow in the low state, the banks will be homogeneous in failure. Given that the regulator cannot observe banks' investment decision, but only realized cash flows, the bailout decision cannot be contingent on the banks' project choice. The bailout probability for each bank will only be contingent on the aggregate state, where aggregate state is defined by the total number of failures in the banking system.

According to Lemma 1, there will be some number of bank liquidations k^*, such that the Regulator is indifferent between liquidating and bailing out the $k^* - th$ bank. Given f bank failed at date 1, k^*banks will be randomly selected and sold to the outside investors, while $f - k^*$will be bailed out. The optimal number of banks to be liquidated, k^*, is defined by the following expression

$$\underline{R} - L\left(k^*\right) = p\left(k^*\right) \tag{5}$$

Since outside investors are rational, and their participation constraint given in (2) holds, k^* is determined by

$$\underline{R} - L\left(k^*\right) = L\left(k^*\right) \tag{6}$$

Lemma 2 *Let k^* be determined by $L(k^*) = \frac{R}{2}$ and $n > k^*$. Then, in any sub-game perfect equilibrium, if f is the number of banks that failed at date 1, Regulator's ex-post optimal bailout policy is defined as follows:*

- *when $f \leq k^*$, all failed banks get liquidated and no bailouts happen*

- *when $f > k^*$, k^* banks are liquidated through the sales to outsiders, while $f - k^*$ banks are bailed out*

From here it is obvious that the pace at which $L(k)$ decreases in the number of banks bought and managed by outsiders will determine how socially beneficial bailouts are or how costly bank liquidations are. If the social loss from liquidation is too high relative to the reduction in deposit insurance costs obtained, this implies that there are 'too many banks to fail' as termed in Acharya and Yorulmazer [2007]. In these cases, I will consider the banking crisis to be systemic.

From Lemma 2, it is clear that bailouts will occur only when enough banks have failed i.e. the banking crisis is systemic. Once enough banks failed together,they would all have the same probability of being bailed out. Ex ante, the bailout probability would be increasing in the total number of failures.

Unless all banks have invested in the market project, the number of bank failures f will have a Binomial distribution $B(n, 1 - \alpha)$ such that probability of f banks failing at date 1 is

$$Pr(f) = C(n, f)\,\alpha^{n-f}\,(1 - \alpha)^f \quad for\, f \in \{0, 1, ...n\} \tag{7}$$

where $C(n, f)$ is the number of combinations of f objects from a total of n.

Therefore, the Regulator's ex-post optimal bailout policy, translates to the following bailout probabilities, where $Pr(Bailout)$ stands for the bailout probability:

- when $f \leq k^*$, $Pr(Bailout \mid f) = 0$

- when $f > k^*$, $Pr(Bailout \mid f) = 1 - \frac{k^*}{f}$

Bank's decision

In the first period all banks are identical and are choosing between the same investment options: invest in the market project, or invest in the bank-specific project. The game between banks is a symmetric game, given that they are identical with the same set of available actions and assumed incapable of distinguishing between the other players (A.W. [1950]).

Therefore, I will be analysing the choice of a representative Bank i which takes the actions of all other banks as given. In order to define a symmetric equilibrium, I will check for profitable deviation of Bank i ,given all other banks are homogeneous in their choice. Since a symmetric equilibrium is defined, if it is profitable for Bank i to deviate, I conclude that all banks will deviate.

When choosing its investment at $t = 0$, Bank i takes the bailout probability as given. In equilibrium each bank correctly infers the ex-post optimal regulator's bailout policy correctly. Therefore, given the bailout policy and implied bailout probabilities, representative Bank i decides between investing in the idiosyncratic $x_i = 1$ and the market project, $x_i = 0$.

Let $E(\pi(0))$ be the expected payoff given all banks invest in the common project. If the low cash flow realizes at date 1, all banks will fail together and the Regulator would randomly pick $n - k^*$ banks to be bailed out, while tliquidate the rest. Thus, the expected payoff from investing in the market project is given as:

$$E(\pi(0)) = \alpha\left(\overline{R} - r\right) + \alpha V + (1 - \alpha)\left(1 - \frac{k^*}{f}\right)\underline{R} \qquad (8)$$

If Bank i thinks that all other banks will choose the market project, it is neccessary to check whether it is profitable for the Bank i to deviate and invest in the bank-specific project. Let $E(\pi_i(1, \underline{0}))$ be the expected payoff of Bank i given it deviates to $x_i = 1$, while all other banks choose $x_{-i} = 0$:

$$E(\pi_i(1, \underline{0})) = \alpha\left(\overline{R} - r\right) + \alpha V + (1 - \alpha)(1 - \alpha)\left(1 - \frac{k^*}{f}\right)\underline{R} \qquad (9)$$

Ex post optimal bailout policy of the Regulator specified in Lemma 2, implies that the bank will always be liquidated when failing on its own, since $k^* \geq 1$ is assumed. and bailed out with some positive probability when failing with other banks.

By comparing (8) and (9), it is clear that Bank i never chooses to deviate when all other banks are investing in the common market project, since the expected bailout subsidy decreases, the lower the probability of the systemic crisis i.e. Bank i failing with other banks. Therefore, the following holds

$$E\left(\pi_i\left(1,\underline{0}\right)\right) - E\left(\pi\left(0\right)\right) = -\alpha\left(1 - \alpha\right)\left(1 - \frac{k^*}{f}\right)\underline{R} < 0$$

Now, I consider the case in which all banks choose to invest in the bank-specific projects. Bank i's expected payoff from investing in the bank-specific project, given all banks invest in their own bank-specific projects is given as:

$$E\left(\pi\left(1\right)\right) = \alpha\left(\overline{R} - r\right) + \alpha V + \left(1 - \alpha\right)\sum_{j=k^*}^{n-1} Pr\left(j\right)\frac{j + 1 - k^*}{j + 1}\underline{R} \qquad (10)$$

Since the projects are uncorrelated, bank failures will be uncorrelated as well. In the states in which very few banks fail together, the Regulator will be selling them to outside investors. In the states in which enough failures occurs simultaneously, some banks will be bailed out.

For $f > k^*$, $f - k^*$ banks will be bailed out at random, while the rest of the failed ones will be liquidated.

In order to determine the equilibrium strategy for the representative Bank i, again the existance of the profitable deviation has to be checked. The expected payoff from invesitng in the market project, while other banks choose the bank-specific projects is given as:

$$E\left(\pi_i\left(0,\underline{1}\right)\right) = \alpha\left(\overline{R} - r\right) + \alpha V + \left(1 - \alpha\right)\sum_{j=k^*}^{n-1} Pr\left(j\right)\frac{j + 1 - k^*}{j + 1}\underline{R} \qquad (11)$$

Deviation payoff in (11) is exactly the same as the payoff in (10) when the Bank i follows other banks' strategy of choosing the bank-specific project. Thus, these two investment strategies are payoff equivalent. Since by choosing the market, when everyone else chooses their own uncorrelated bank-specific projects, the bank i is still uncorrelated with other banks, exactly like when choosing its own bank-specific project.. Given all projects are identical in terms of cash flows and success probabilities, then, for the same level of interbank correlation, payoffs from different strategies could be the same. Thus, there might be multiple

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Nash equilibria resulting in the same level of interbank correlation which are all payoff-equivalent.

Here I want to focus on the symmetric equilibria in which all banks take the same equilibrium action. Therefore, for the ex-post optimal bailout policy of the Regulator, banks always choose to invest in the market project. By herding, they achieve perfect interbank correlation.. This is in order to maximize the bailout subsidies by maximizing the probability of bailouts happening. This is achieved by increasing the probability of the systemic crisis through perfectly correlating the investment portfolios.

Lemma 3 *Let $E\left(\pi\left(0\right)\right)$ be defined as in (8) and $E\left(\pi\left(1\right)\right)$ as in (10). If the Regulator is implements the ex post optimal bailout policy, banks will always choose to invest in the common market project, rather than in the bank-specific projects. This is due to $E\left(\pi\left(0\right)\right) > E\left(\pi\left(1\right)\right)$.*

Proof: See appendix.

Ex-post optimal bailout policy of the Regulator, defined in Lemma 2, combined with the optimal investment decision of the banks gives the subgame perfect equilibrium described in Proposition 1.

Proposition 1 *Let k^{*} be determined by $L\left(k^{*}\right) = \frac{R}{2}$. Then, in the unique subgame perfect equilibrium:*

If f is the number of failed banks at date 1, and $f > k^{*}$, the Regulator will liquidate at maximum k^{*} banks and bailout the rest of the failed ones, such that the bailout probability for each bank is equal across all failed banks.

Then, the optimal investment choice for each bank is to invest in the common market project and therefore maximize the expected bailout subsidy by maximizing the interbank correlation.

4 'Fail less' Model - Heterogeneity in Failure

In order to investigate how the banks' investment decision and choice between systematic and idiosyncratic risk would be affected by introducing the Regulator who cares to bailout banks that failed less, I introduce some heterogeneity in banks' failure.

In case of bank failure, the second-period cash flows are a function of the realized first-period cash flows as defined in (1). Thus, heterogeneity in failure is introduced through heterogeneity in projects' cash flows in the low state. The bank-specific projects will remain homogeneous and uncorrelated across banks, but will have a higher cash flow than the market project, in the low state:

$$\underline{R}_i > \underline{R}_m$$

In the high state, the cash flows will remain the same across projects, i.e. $\overline{R}_i = \overline{R}_m$ still holds.

In this setup, how bad the bank failed at date 1, would determine the cash flows its project can yield in the second period, i.e. continuation value of the bank in failure. Consequently, the Regulator will no longer be indifferent towards which banks to bailout and which ones to liquidate. Heterogeneity of banks in failure will affect the total banking output in the second period.

The Regulator's optimal bailout decision would have to determine the total number of banks to be bailed out, but also which banks to bailout and which ones to liquidate, given their value in failure.

4.1 Solving the Model

Regulator's decision

At date 1, for each bank that failed, the Regulator compares payoffs from bailing out and liquidating the bank, given the total number of failures in the banking system. The banks will be heterogeneous in failure, since now the market and bank-specific projects no longer have the same cash flow in the low state. The social loss of liquidation will now depend on the extent to which the bank has failed i.e. whether it invested in the bank-specific or the market project.

Since the Regulator is maximizing the total banking output at date 2, net of the costs of his intervention, it follows that the bailout probability for each bank will be contingent on the aggregate state, where aggregate state is defined by the total number of failures in the banking system, but also on the bank's value (cash flow realized) in failure.

Let $j = i, m$ then for any bank that failed, regardless of the project in which it invested, the payoff from bailing out Bank i is given as:

$$E\left(R_{2j} \mid \underline{R}_{1j}\right) - \left(r - \underline{R}_{1j}\right) = \underline{R}_{1j} - \left(r - \underline{R}_{1j}\right)$$

On the other hand, payoff from liquidating the Bank i, where the bank is $k - th$ bank to be liquidated is:

$$L\left(k\right) - \left(r - \underline{R}_{1j} - p\left(k\right)\right) = L\left(k\right) - \left(r - \underline{R}_{1j} - L\left(k\right)\right)$$

By comparing these payoffs we can see that the social loss from selling the k-th Bank is

$$\underline{R}_j - L\left(k\right) \tag{12}$$

while the gain from selling the k-th Bank is $p\left(k\right)$ which represents the reduction in deposit insurance cost, the price outsiders pay per bank, given k banks are sold in total.

Similar to the benchmark model, there will be some number of bank liquidations k_j^*, such that the regulator is indifferent between liquidating and bailing out the $k_j^* - th$ bank, where $j = i, m$. Since now there are two possible realizations of cash flow $\underline{R}_i$ and $\underline{R}_m$ in failure, the Regulator will have two different thresholds for the number of banks he chooses to liquidate, depending on whether liquidated banks invested in bank-specific or the market project.

For the banks that invested in the common project, the Regulator will be indifferent between liquidating and bailing out the $k_m^* - th$ bank, where k_m^* is implicitly defined as

$$\underline{R}_m - L\left(k_m^*\right) = p\left(k_m^*\right) \tag{13}$$

Similarly, for the banks that invested in the bank-specific projects, the Regulator will be indifferent between liquidating and bailing out the $k_i^* - th$ bank, where k_i^* is implicitly defined as

$$\underline{R}_i - L\left(k_i^*\right) = p\left(k_i^*\right) \tag{14}$$

Outsiders are rational, and their participation constraint binds, thus

$$p\left(k_j^*\right) = L\left(k_j^*\right)$$

holds for any k_j^*.

Since the loss from liquidation increases in the bank's expected value of the second-period cash flows, it is more costly to liquidate higher value banks, This results in the smaller number of bank liquidations among the banks that will yield higher second-period cash flows conditional on bailout.

Consequently, if the Regulator observes bank failures across both banks that invested in the market and idiosyncratic projects, he will always first choose to liquidate the lower-value banks i.e. banks that invested in the market project.

Lemma 4 *Let bank-specific projects have higher cash flow than the market project in case of failure, i.e. $\underline{R}_i > \underline{R}_m$, and k_m^* and k_i^* represent the maximum number of banks liquidated that invested in the market project, and that invested in the bank-specific project.*

In equilibrium, the number of liquidated banks that invested in the bank-specific projects will be lower than the number of liquidated banks that invested in the market project i.e. $k_m^* > k_i^*$ holds.

Proof: Since $\underline{R}_i > \underline{R}_m$, from (13) and (14) and outsiders participation constraint, it follows that $L\left(k_i^*\right) > L\left(k_m^*\right)$. Given $L\left(k\right)$ is decreasing in k, it follows that $k_m^* > k_i^*$.

The price outsiders pay is only contingent on the total number of banks they purchase while independent of banks' values. On the other hand, the losses from bank liquidation will be increasing in the bank's value which creates incentives for the Regulator to preserve the higher value banks in the banking system, while liquidating the lower valued ones first.

Whenever the crisis is systemic enough for bailouts to happen and among the failed banks there are some that invested in the bank-specific projects, while others invested in the market project, the banks that failed less will have higher probability of being bailed out. Ex-post optimal bailout policy for the Regulator will depend on the total number of failures. If not enough banks failed at the same time, crisis is not systemic enough and even the high value banks can

be sold to outsiders. Also, the bailout policy will be such that the bailout probability will be increasing in the bank's value in failure. This is the key feature of the ex post optimal bailout policy, here termed 'fail less' bailout policy.

Proposition 2   *Let k_j^* be i determined by $L\left(k_j^*\right) = \frac{R_j}{2}$, where $j = i, m$. Then in any sub-game perfect equilibrium, Regulator's ex post optimal bailout policy is defined as follows:*

If $n > k_i^*$ and f_j is the total number of banks that failed at date 1 while investing in project $\widetilde{R}_j$, then:

- *when $f_j \leq k_j^*$, all failed banks f_j get liquidated and no bailouts happen*

- *when $f_j > k_j^*$, k_j^* banks are liquidated through the sales to outsiders, while $f_j - k_j^*$ banks are bailed out*

Then, Regulator's ex-post optimal bailout policy translates into corresponding bailout probabilities as defined in Corollary 1.

Corollary 1   *Let $f = f_i + f_m$, where f_m is the number of failed banks that invested in the market and f_i is the number of failed banks that invested in the bank-specific project. Depending on the total number of failures, and their investment decision, the banks will face different bailout probabilities, given the Regulator implements his ex-post optimal bailout policy:*

1. *When $f \leq k_i^*$ no bailouts happen, and each failed bank gets liquidated regardless of its investment project*

2. *If $f > k_i^*$, such that*

   - *$f_m \leq k_i^*$, then all f_m failed banks get liquidated, so that $Pr\left(Bailout \mid f_{i,m}, \tilde{R}_m\right) = 0$, while $k_i^* - f_m$ of f_i banks get liquidated , so that $Pr\left(Bailout \mid f_{i,m}, \tilde{R}_i\right) = \frac{f_i - (k_i^* - f_m)}{f_i} = \frac{f - k_i^*}{f_i}$*

   - *$k_i^* \leq f_m < k_m^*$, then all f_m banks get liquidated, so that $Pr\left(Bailout \mid f_{i,m}, \tilde{R}_m\right) = 0$, while all f_i banks gets bailed out, so that $Pr\left(Bailout \mid f_{i,m}, \tilde{R}_i\right) = 1$*

- $f_m > k_m^*$, then k_m^* out of f_m banks gets liquidated, while the rest of the failed banks are bailed out, so that bailout probabilities are $Pr\left(Bailout \mid f_{i,m}, \tilde{R}_m\right) = \frac{f_m - k_m^*}{f_m}$, and for banks that invested in the idiosyncratic project $Pr\left(Bailout \mid f_{i,m}, \tilde{R}_i\right) = 1$

Bank's decision

As in the benchmark model, banks are identical in the first period and they choose between investing in the market or in the bank-specific project. Following the definition and assumptions of symmetric games, as before, I will analyse the investment choice of a representative Bank i which takes the actions of all other banks as given.

If all banks choose to invest in the market project, then if low cash flow realizes at date 1, all banks fail together. Given the bailout policy given in Proposition 2, the bailout probability for each bank will be $1 - \frac{k_m^*}{n}$ Thus, the expected payoff from investing in the market project is given as:

$$E\left(\pi\left(0\right)\right) = \alpha\left(\overline{R}_m - r\right) + \alpha V + (1 - \alpha)\left(1 - \frac{k_m^*}{n}\right)\underline{R}_m \tag{15}$$

If Bank i deviates and chooses the bank-specific project, while others invest in the market, the Bank's failure is no longer correlated with other banks' failures. Given $k_i^* \geq 1$, the bank will always be liquidated when failing on its own. However, if the low cash flow of the idiosyncratic project realizeS, when the market project also yields the low cash flow, Bank i will be bailed out with certainty, as it will be the bank with the highest value in failure.

Thus, the expected payoff given Bank i deviates to $x_i = 1$, while all other banks choose $x_{-i} = 0$ is the following:

$$E\left(\pi_i\left(1,\underline{0}\right)\right) = \alpha\left(\overline{R}_i - r\right) + \alpha V + (1 - \alpha)(1 - \alpha)\cdot 1 \cdot \underline{R}_i \tag{16}$$

If the payoff from deviating is high enough, the bank will choose the idiosyncratic project, even when all other banks invest in the common project. Therefore, the condition that should hold is:

$$E\left(\pi_i\left(1,\underline{0}\right)\right) - E\left(\pi\left(0\right)\right) > 0$$

which translates into

$$(1 - \alpha) \underline{R}_i > \left(1 - \frac{k_m^*}{n}\right) \underline{R}_m \tag{17}$$

Condition (17) implies that, given failure, expected bailout subsidies have to be higher when the Bank i chooses the bank-specific project, than when it chooses the common project. From (17), I calculate the α_{FL}^* , the probability of projects yielding high cash flows, that makes the Bank i indifferent between the two projects, when all other banks invest in the market:

$$\alpha_{FL}^* = 1 - \left(1 - \frac{k_m^*}{n}\right) \frac{\underline{R}_m}{\underline{R}_i} \tag{18}$$

Then, if $\alpha < \alpha_{FL}^*$, it is profitable for Bank i to invest in the bank-specific project, even if all other banks choose the common project. Since banks are identical, this implies that in that case, all banks would choose to deviate and invest in the bank-specific project.

In case all banks choose to invest in the bank-specific projects, the expected payoff $E(\pi(1))$ is

$$E(\pi(1)) = \alpha\left(\overline{R}_i - r\right) + \alpha V + (1 - \alpha) \sum_{f=k_i^*}^{n-1} Pr(f) \frac{f + 1 - k_i^*}{f + 1} \underline{R}_i \tag{19}$$

Since bank-specific projects are uncorrelated, bank failures will be uncorrelated as well. If only few banks fail, Regulator can always liquidate them at high enough price, such that the opportunity cost of bailout is too high.

It is clear that for Bank i it would never be profitable to deviate and invest in the market project, when all other banks are investing in their bank-specific projects, since its expected bailout subsidy in this case would be zero.

Since bank-specific projects give higher cash flows in failure than the common project, a bank that invested in the common project would experience the most severe faiure and the Regulator would always liquidate it, given all other banks invested in the market project. If failing on its own, the price outsiders are willing to pay is high enough, so no bailouts will happen in general. If failing with others such, that crisis is systemic enough, Regulator will always liquidate

the bank with the lowest value in failure first, while bailout the banks with higher values.

Therefore, under these assumptions, if Bank i believes that all other banks will choose to invest in the bank-specific projects, it will be optimal for Bank i to invest in its bank-specific project as well, since that would maximize the expected bailout subsidies.

Optimal investment strategy for banks, given the Regulator implements ex post optimal 'fail less' bailout policy is summarized in Proposition 3.

Proposition 3 *Let α_{FL}^* be defined as in (18) and the ex post optimal bailout policy of the Regulator be defined as in Proposition 2.*

- *If $\alpha < \alpha_{FL}^*$ holds, in equilibrium all banks choose to invest in their bank-specific projects $x^* = 1$*

- *If $\alpha \geq \alpha_{FL}^*$ holds, in equilibrium all banks choose to invest in the market project $x^* = 0$*

'Fail less' revisited - robustness check

Here, I try to separate the effects of the bailout probability channel from the project cash flow channel, on the banks' investment decision. In order to show that banks' decision to invest in the idiosyncratic project is not purely driven by the assumption that the idiosyncratic project has higher expected return, but that 'fail less' bailout policy incentivizes banks to differentiate in failure, I try to isolate the bailout probability effect on the banks' investment decision.

Higher expected value project may be preferred byh banks, even if bailouts were random and cash flows in failure had no effect on the bailout probability. However, bailout probability channel captures the effect of the higher bailout probability given failure, if the bank chooses to invest in the better project, without even capturing the higher cash flows of this better project. In this analysis, better project is the one that yields higher cash flows in the bad state i.e. in failure.

I show that the result that banks choose to herd less because the bailout probability increases in the bank's value in failure, still remains, even when banks are

not allowed to capture the cash flow benefits from choosing the higher-payoff project.

In order to show this, I assume that if bailed out the banks will only receive some V_b from the continuation cash flows, while the regulator will keep the rest i.e. $\underline{R}_j - V_b$ where $j = i, m$. I assume that $V_b < \underline{R}_m < \underline{R}_i$ holds.

Solving for the banks' optimal investment decision, given Regulator's ex-post optimal bailout policy, as defined in Proposition 2, I obtain the following:

By comparing the expected bailout subsidies of two investment possibilities, which here differ only through the probability of bailout, itt can be derived that, for any Bank i it is profitable to deviate and invest in the bank-specific project, as long as $\alpha < \alpha_b^*$ holds, where α_b^* is defined as follows:

$$\alpha_b^* = \frac{k_m^*}{n} \tag{20}$$

The larger k_m^* implies that it is less costly to liquidate banks that invested in the common project, therefore more banks can be liquidated. This makes investment in the bank-specific project more attractive. When the number of liquidations is large among banks that choose the market project, not even the higher probability of state in which bailouts happen is enough to compensate for the lower probability of bailout for each bank.

When banks receive the same cash flow i.e. continuation value after the bailout, regardelss of their investment decision, this reduces the incentives to invest in the idiosyncratic project coming from the cash flow channel. The only incentive that remains is the higher bailout probability given failure, since the Regulator still prefers to save the banks that failed less.

Lemma 5 Let α_b^* be defines as in (20) and α_{FL}^* be defined as in (18). Then $\alpha_b^* < \alpha_{FL}^*$ holds.

Proof: See Appendix.

The bailout probability channel results in the higher bailout probability for banks that failed less, which still induces banks to herd less. However, the range of parameter values for which no herding is optimal will be smaller than when bailout probability channel works together with the cash flow channel i.e. banks are able to capture higher continuation values from investing in the idiosyncratic projects.

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5   Welfare Analysis

In order to analyse which investment policy implemented by banks maximizes the social welfare, I calculate the total expected banking output net of any costs related to potential interventions of the Regulator, for different investment choices banks can make. This allows me to discuss the benefits and time-consistency of the Regulator's bailout policies.

It is very often the case that Regulator's ex ante policy is not what it would be the optimal bailout policy ex post. Namely, even if a Regulator could credibly commit to some ex ante optimal policy that induces the socially optimal investment behaviour of banks, it might be ex post optimal for the Regulator to deviate from his commitment. Therefore, having the bailout policy that is time consistent could be very important for being able to implement the socially preferred outcome and affect the banks' investment decision in a positive way.

By analysing social welfare, I find that the ex post optimal 'fail less' bailout policy implements the ex ante optimal investment outcome.

First, I calculate the total expected banking output net of intervention costs, given all banks choose to invest in the common project. Let $E\left(\Pi\left(0\right)\right)$ represent the ex-ante welfare under the common project investment choice for all banks.

$$
\begin{aligned}
E\left(\Pi\left(0\right)\right) \;=\;\; & n \cdot \alpha \left(\overline{R}_m + V\right) + n \cdot \left(1-\alpha\right) \underline{R}_m \\
& - k_m^* \left(1-\alpha\right) \left(\underline{R}_m - L\left(k_m^*\right)\right) \\
& - \left(1-\alpha\right) \left(n\left(r - \underline{R}_m\right) - k_m^* \cdot p\left(k_m^*\right)\right) \qquad (21)
\end{aligned}
$$

The first row of equation (21) represents the total expected banking output in high cash flow realization. The second row represents the social loss arising from bank liquidations, given k_m^* banks get liquidated, and outsiders are able to produce only $L\left(k_m^*\right)$ of the banks' continuation value. The third row in (21) represents deposit insurance cost, which is equal to the amount Regulator has to cover due to n banks failing with probability $\left(1-\alpha\right)$. The cost of deposit insurance is reduced by the amount of liquidity Regulator collects through bank liquidations, and that is in equilibrium $k_m^* \cdot p\left(k_m^*\right)$.

In equilibrium, outsiders will be paying a fair price for the banking assets they

purchase, such that (14) holds. Therefore, it would be that $\underline{R}_m = 2L(k_m^*)$ and equation (21) simplifies to:

$$E\left(\Pi\left(0\right)\right) = n \cdot \alpha \left(\overline{R}_m + V\right) + n \cdot \left(1 - \alpha\right) \underline{R}_m - \left(1 - \alpha\right) \left(n \left(r - \underline{R}_m\right)\right) \quad (22)$$

It is clear from (22) that the social loss from bank liquidation is perfectly offset by the decrease in the costs of deposit insurance obtained through bank sales. Therefore, the social welfare is equal in terms of value to the situation in which the Regulator is bailing out all banks that failed, given they all invested in the common project.

Next, I consider the case in which all banks invest in the bank-specific projects. From Proposition 2, it follows that the banking crisis will be systemic enough for the Regulator to intervene, only when more than k_i^* banks have failed simultaneously. Therefore, the social welfare is given by:

$$
\begin{aligned}
E\left(\Pi\left(1\right)\right) \;\; = \;\; & \sum_{f=0}^{n} Pr\left(f\right)\left(n - f\right)\left(\overline{R}_i + V\right) \\
& + \sum_{f=0}^{k_i^*-1} Pr\left(f\right)\left(f \cdot \left(L\left(f\right) - \left(r - \underline{R}_i - p\left(j\right)\right)\right)\right) \\
& + \sum_{f=k_i^*}^{n} Pr\left(f\right)\left(jfR - k_i^*\left(\underline{R}_i - L\left(k_i^*\right)\right) - \left(f\left(r - \underline{R}_i\right) - k_i^* \cdot p\left(k_i^*\right)\right)\right) \quad (23)
\end{aligned}
$$

Given that the expected number of failed banks $\sum_{j=0}^{n} jPr(j)$ represents the expected value of binomially distributed variable $j \sim B\left(n, 1 - \alpha\right)$, it follows that

$$\sum_{j=0}^{n} jPr(j) = n\left(1 - \alpha\right)$$

holds. Then the first term of (23) is transformed into:

$$\left(\overline{R}_i + V\right)\left(n - n\left(1 - \alpha\right)\right) = n\alpha\left(\overline{R}_i + V\right) \quad (24)$$

Since idiosyncratic and market project yield the same cash flows in the high

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state, the expected banking output of banks that did not fail is independent of the investmen choice, as seen in (24).

When banks invest in their bank-specific projects, potential bank failures are uncorrelated. Therefore, in the states in which only few banks fail, the Regulator will be able to liquidate the banks such that the social loss from liquidation is smaller than the reduction in the cost of deposit insurance.

The second row of the equation (23) represents the banking output from liquidations net of the costs of deposit insurance which arises in states in which there is still too few banking failures for the bailouts to happen. Since the number of bank failures is low enough, the gains from selling the banking assets to outsiders outweigh the social loss from the bank liquidations. Therefore, in these states the Regulator is able to capture the additional banking output. Conversly, when all banks fail together the gains from bank sales is always offset by the social loss from liquidation.

Detailed derivations being provided in the Appendix, I obtain the total welfare when banks invest in the bank-specific projects to be

$$
\begin{aligned}
E\left(\Pi\left(1\right)\right) \;=\;\; & n\alpha\left(\overline{R}_i + V\right) + \left(1-\alpha\right)n\underline{R}_i \\
& + \sum_{f=0}^{k_i^*} f\, Pr\left(f\right)\left(2L\left(f\right) - \underline{R}_i\right) \\
& - \left(1-\alpha\right)n\left(r - \underline{R}_i\right)
\end{aligned}
\tag{25}
$$

The first row of (25) represents the expected banking output of the high and low cash flow realizations. The second row captures the the gains Regulator is able to capture from bank liquidations in the states of the world in which there is few enough bank failures, so such that the gains from bank sales are not entirely offset by the social loss from bank liquidation When less than k_i^* banks fail together, gains from bank sales are larger than the social loss arising from outsiders being less efficient in managing the banking assets. Therefore, when the number of liquidated banks j is $j \in [0, k_i^*)$:

$$
\underline{R}_i - L\left(j\right) < p\left(j\right)
\tag{26}
$$

holds. In addition, the participation constraint of the outsiders holds for any j:

$$p(j) = L(j) \tag{27}$$

By combining (26) and (27), I obtain: $2L(j) > \underline{R}_i$ for every $j \in [0, k_i^*)$.

When there is enough bank failures, such that the crisis is systemic enough for the Regulator to start bailing out banks, this implies that the gains from bank sales are entirely offset by the social loss from bank liquidation.

I define the additional welfare arising from bank investments being uncorrelated when they invest in the bank-specific projects, so called gains from no herding, by β_{FL}. This additional welfare arises from the fact that the systemic banking crisis arises less often than when banks' investments are perfectly correlated. This implies that the Regulator is able to capture additional gains from selling banks at higher prices, and so reducing the cost of deposit insurance for failed banks.

The gains from no herding β_{FL} are always strictly positive and defined as

$$\beta_{FL} = \sum_{j=0}^{k_i^*} j Pr(j)(2L(j) - \underline{R}_i) \tag{28}$$

Then (25) can be written as

$$E(\Pi(1)) = n\alpha\left(\overline{R}_i + V\right) + (1 - \alpha)n\underline{R}_i - (1 - \alpha)n\left(r - \underline{R}_i\right) + \beta_{FL} \tag{29}$$

In order to determine the investment policy of banks that maximizes the social welfare, I compare the welfare of this economy given all banks choose the common project with the welfare when all banks choose thier bank-specific projects, as given in equations (22) and (29).

The optimal investment would be the one yielding maximum total banking output net of all the costs that may occur due to Regulator's interventions.

In the economy where $\overline{R}_i = \overline{R}_m$, the following always holds

$$E(\Pi(1)) - E(\Pi(0)) > 0$$

since $\beta_{FL} > 0$ always holds.

Therefore, the Regulator prefers to implement the bailout policy which would incentivize the banks to invest in their bank-specific projects and thus minimize the level of correlation between their investment portfolios.

Proposition 4 *The expected total output of the banking sector at date* 0, *net of any costs of anticipated liquidations and bailouts, is maximized when banks choose uncorrelated investment projects, that is, when all banks invest in their own bank-specific project.*

Bailout policy that implements optimal investment outcome and is at the same time ex post optimal, i.e. the Regulator will not deviate once bank failures occur, can be considered optimal and time-consistent bailout policy.

From the model analysed earlier, it can be said that the ex post optimal 'fail less' bailout policy , is also the bailout policy that induces banks to choose ex ante optimal investment. In such way banks choose the investment project in anticipation of the 'fail less' bailout policy being implemented and manage to maximize the total expected banking output net of any anticipated bailout and liquidation costs.

In order to provide more general implications on welfare and optimal bailout policy, in the next section I analyse the economy with heterogeneous banks, endowed with different bank-specific projects, some yielding lower and some higher failure cash flows than the market project cash flows.

6   Heterogeneous banks

In order to obtain a more general result on how the bailout policy that accounts for banks' value in failure affects banks' investment decision, it is neccessary to turn to the more realistic setup in which the banking system is populated by heterogeneous banks. Namely, until now the change in investment decision and the type of risk banks choose to undertake has been analyzed in the economy consisting only ofk homogeneous banks that were all endowed with the idiosyncratic project that was strictly better than the market project in failure. In order to obtain the heterogeneity, I allow for the idiosyncratic projects of banks to be different in their low state cash flow realizations.

Here, I introduce two types of banks, Good banks endowed with the idiosyncratic project $\widetilde{R}_G$, and Bad banks endowed with the idiosyncratic project $\widetilde{R}_B$.

Both bank types also have access to the common market project $\tilde{R}_m$ defined as before. All project cash flows are defined as before, in Section 2 describing the homogeneous case, with the following relation holding between different project cash flows:

$$\bar{R}_m \quad = \quad \bar{R}_G = \bar{R}_B$$
$$r > \quad \underline{R}_G > \quad \underline{R}_m > \underline{R}_B \tag{30}$$

Relationship given in (30) shows that in case of the high state realization all projects are the same, and heterogeneity in idiosyncratic projects only matters when the low cash flows realize. Therefore the Bad banks are endowed with the idiosyncratic project that yields the lowest cash flows relative to the market and Good banks' idiosyncratic project low state. This implies that when low cash flow realizes, any bank will fail since none of the projects has low cash flows high enough to cover the promised return to depositors.

Apart from introducing two bank types through two different idiosyncratic investment projects that are heterogeneous in the low state, all other components of the model are preserved.

At date 0 banks are choosing to invest their depositors' endowement in the idiosyncratic ($x_i = 1$) or in the market project ($x_i = 0$). Since now there are two types of banks, but banks are identical within the type, the heterogeneous banks game will be solved for the representative bank of each type. In other words, Good banks may be investing differently than the Bad banks, but all banks within the type will be investing in the same way. Given that the economy is populated by n banks in total, now there will be g Good banks and b Bad banks, such that $g + b = n$.

If the low cash flows realize, the bank is not able to meet its liabilities towards depositors and is considered failed. The Regulator has to decide between selling the failed bank to outside investors, or bailing out the bank, while covering the deposits in both cases. As before the Regulator is maximizing the total banking output net of any costs of his intervention, and therefore implementing the ex post optimal 'fail less' bailout policy, as described in the homogeneous case.

Outside investors are willing to purchase the failing bank assets, as long as they break even. Similar to the homogeneous case, outside investors are less efficient

users of the banking assets and the liquidation value they are able to produce is a decreasing function of the total number of failed banks that Regulator chooses to liquidate. In return for selling the bank to outsiders, the Regulator receives some additional liquidity in terms of the price outsiders are willing to pay for the assets they are purchasing. This additional liquidity reduces costs of deposit insurance which Regulator has to incur on behalf of all the failing banks. This implies that the Regulator will again be optimizing such that bank sales to outsiders happen until the gains from reduced cost of deposit insurance are exactly offset by the social loss of bank liquidation.

The novelty of the heterogeneous case is that now there will be different thresholds in terms of number of failing banks Regulator is willing to sell, depending on the bank type and investment the banks have undertaken.

Given that now the banks are no longer the same, different bank types may end up undertaking different risks from the perspective of idiosyncratic vs. systematic risk.

I solve the model for the heterogeneous bank case, with two bank types starting from the date 1 and the Regulator's subgame.

Regulator's decision

Given that in total there are three types of investment projects in this economy, the market, the good and the bad idiosyncratic project, there could be three different low cash flow realizations in bank failure. For each bank that failed, Regulator will compare the payoff from bailing the bank out and the payoff from bank liquidation, taking into account the total number of banks to be liquidated which will determine the liquidation value and the liquidation price outside investors are willing to pay.

Let j determine the project the bank has undertaken, where $j = G, B, m$. Then, for each project there will be some number of bank liquidations k_j^* such that the regulator is indifferent between liquidating and bailing out the $k_j^* - th$ bank that has invested in $\widetilde{R}_j$ project at date 0. Consequently, for each j, the following will hold:

$$\underline{R}_j - L\left(k_j^*\right) = p\left(k_j^*\right) \tag{31}$$

Also, the break even condition for the outsiders holds. Then, k_j^* is implicitly defined by $\underline{R}_j = 2L\left(k_j^*\right)$.

Given the relation between cash flows stated in (30) holds, and $L\left(k\right)$ and $p\left(k\right)$ are decreasing in k, then:

$$k_B^* > k_m^* > k_G^* \tag{32}$$

This implies that the lower cash flow of the project bank has undertaken is, it will be less costly to liquidate these banks in failure. Therefore, the number of liquidated failed banks before any bailouts happen will be the highest among the bad banks that invested in the Bad idiosyncratic project, while the lowest among the good banks that invested in the Good idiosyncratic project.

Since the price outsiders pay is only contingent on the total number of banks that are being sold, while the liquidation loss is increasing in the bank's value in failure, the Regulator will always liquidate the low valued banks first, while bailout the higher valued ones, if any bailouts are to happen. For bailouts to happen, the crisis would have to be systemic enough i.e. enough banks have to fail together. How systemic the crisis should be for the Regulator to intervene would depend on the investment project the banks have undertaken and the number of banks with the same project that failed together, as well as the total number of failed banks. Proposition 5 defines the bailout policy of the Regulator in the heterogeneous banks setup.

Proposition 5

Let f_j be the total number of failed banks that invested in project j where $j = G, B, m$. Then, let $f = \sum_j f_j$ be the total number of all failed banks. Then, in any sub-game perfect equilibriu, regulator's ex-post optimal bailout policy is defined as follows:

- *If $f < k_G^*$, no bailouts happen at all.*

- *If $f_B > k_B^*$, then k_B^* of the failed bad banks that invested in the project B will be liquidated, while $f_B - k_B^*$ will be bailed out. In all other possible outcomes any failed bank that invested in project B will be liquidated.*

- *For any f and f_j realized, liquidations start with the lowest valued banks until the Regulator's indifference condition given in (31) is satisfied for the last marginal bank to be liquidated. The rest of the failed banks with higher or equal values in failure are bailed out.*

Good bank that invested in project G will always be bailed out with some probability, unless the total number of all bank failures is lower than k_G^*. The bad bank that invested in its own idiosyncratic project B will have some positive probability of being bailed out only when total number of bad banks that failed is greater than k_B^*.

Such bailout policy translates into different bailout probabilities, depending on the bank type and investment project undertaken, which banks will account for when making their investment decision.

Given that the banks within the type are identical in the first period and facing the same choice at date 0, I will analyse the investment choice of a representative bank of each type, representative Bad bank and the representative Good bank.

Bad Bank's decision

Let $E\left(\pi_B\left(x_B, x_G\right)\right)$ be the expected payoff from the investment project over two periods of the Bad bank, given bad bank chooses x_B, while the Good bank chooses investment x_G. Bad bank is trying to maximize the expected payoff by taking into account Regulator's optimal bailout policy and potential investment choice of the Good banks.

If Bad banks believe that Good banks would choose to invest in the common market project, then the expected payoffs from investing in the common project or investing in the idiosyncratic Bad project are given as follows:

$$E\left(\pi_B\left(0,0\right)\right) = \alpha\left(\bar{R}_m - r\right) + \alpha V + (1-\alpha)\left(1 - \frac{k_m^*}{n}\right)\underline{R}_m \qquad (33)$$

$$E\left(\pi_B\left(1,0\right)\right) = \alpha\left(\bar{R}_B - r\right) + \alpha V + (1-\alpha)\,Pr\left(f_B \geq k_B^*\right)Pr\left(Bailout \mid f, \tilde{R}_B\right)\underline{R}_B \qquad (34)$$

When Good banks are choosing the market project, mimicking the Good banks enables the Bad banks to face the same bailout probability in failure as the

Good banks, therefore this expected payoff corresponds to the homogeneous case, since all banks choose the same project.

In equation (34) the Bad bank that failed while investing in the idiosyncratic project B gets bailed out only when enough of Bad banks have failed together, regardeless of what happened with the good banks, namely $f_B > k_B^*$ has to hold. Then, for a given f_B the bailout probability is equal to:

$$Pr\left(Bailout \mid f, \tilde{R}_B\right) = 1 - \frac{k_B^*}{f_B}$$

If Bad banks believe that Good banks would choose to invest in their idiosyncratic project G, then the expected payoffs from investing in the market project or investing in the B project are given as follows:

$$E\left(\pi_B\left(0,1\right)\right) = \alpha\left(\bar{R}_m - r\right) + \alpha V + (1-\alpha)\left(1 - \frac{k_m^*}{b}\right)\underline{R}_m \qquad (35)$$

$$E\left(\pi_B\left(1,1\right)\right) = \alpha\left(\bar{R}_B - r\right) + \alpha V + (1-\alpha)\,Pr\left(f_B \geq k_B^*\right)Pr\left(Bailout \mid f, \tilde{R}_B\right)\underline{R}_B \qquad (36)$$

Since the idiosyncratic project of Bad banks is dominated by the common market project, it is clear that when choosing the idiosyncratic project Bad banks can be bailed out only when there is enough of project B failures, and therefore payoff for the Bad banks from choosing B does not depend at all on what Good banks will invest in. The following lemma describes the best response of the Bad banks given the investment decision of the Good banks.

Lemma 8

Given Regulator's optimal 'fail less' bailout policy, it is a Bad banks' dominant strategy to invest in the common market project and therefore undertake systematic risk.

Proof: See Appendix.

Good Bank's decision

Given that investing in the market project is a dominant strategy for bad banks, representative Good bank will decide on the optimal investment project taking into account Regulator's optimal 'fail less' bailout policy, and the fact that Bad banks will be investing in the market project. Let $E\left(\pi_G\left(x_B, x_G\right)\right)$ be the expected payoff from the investment project over two periods of the Good bank, given bad bank chooses x_B , while the Good bank chooses investment x_G.

Then, the expected payoffs from investing in the common project or investing in the idiosyncratic project G, when bad banks choose the common project are given as follows:

$$E\left(\pi_G\left(0,0\right)\right) = \alpha\left(\bar{R}_m - r\right) + \alpha V + (1-\alpha)\left(1 - \frac{k_m^*}{n}\right)\underline{R}_m \qquad (37)$$

$$E\left(\pi_G\left(0,1\right)\right) = \alpha\left(\bar{R}_G - r\right) + \alpha V + (1-\alpha)\left[\alpha Pr\left(f_G \geq k_G^*\right)Pr\left(Bailout \mid f_G, \tilde{R}_G\right) + (1-\alpha)\cdot 1\right]\underline{R}_G \qquad (38)$$

From equation (38) it follows that the expected bailout subsidy in case the Good bank that invested in the idiosyncratic project G failed, consists of two parts. First, expected bailout subsidy in case the bad banks that invested in the market project did not fail, depends on whether enough of good banks have failed together, so that crisis can be considered systemic among the good banks and that any bailouts would happen. The crisis among the good banks is considered systemic when there is more than k_G^* failures of good banks. Second, in case the market project yielded low cash flow, so that all bad banks failed simultaneously, this would create enough of the systemic crisis for the regulator to intervene, and then any failed good bank would be bailed out with certainty.

By comparing the expected payoffs of two investment strategies given in (37) and (38), I can define the optimal investment strategy for Good banks, given Bad banks' optimal investment and anticipated Regulator's optimal bailout policy.

Two investment projects differ only in the payoff given bank failure, that is the bailout subsidy. Thus, I compare the bailout subsidies in order to define optimal investment policy of the Good bank.

Lemma 9

In equilibrium, the Good banks will choose to invest in the idiosyncratic project G, $x_G = 1$, when $\alpha < \alpha^*_{HFL}$ holds and α^*_{HFL} is defined as follows:

$$\alpha^*_{HFL}\left(1 - \Theta\left(\alpha^*_{HFL}\right)\right) = 1 - \left(1 - \frac{k^*_m}{n}\right)\frac{R_m}{\underline{R}_G} \tag{39}$$

where $\Theta\left(\alpha^*_{HFL}\right) = \sum\limits_{f_G = k^*_G}^{g-1} Pr\left(f_G\right)\left(1 - \frac{k^*_G}{f_G + 1}\right)$, and f_G is the number of good banks that failed at date 1 and $f_G \sim B\left(g, 1 - \alpha\right)$.

If $\alpha \geq \alpha^*_{HFL}$, Good banks will choose to invest in the market project, thus undertaking the systematic risk and herding with the Bad banks.

Proof: See Appendix.

From Lemma 9 it is clear that once heterogeneous banks exist, there is no more multiple equilibria for $\alpha \geq \alpha^*_{HFL}$ range of parameters, as it was the case in the homogeneous banks setup. When α is high enough, it is optimal for the Good bank to pick the market project, though other good banks may be choosing the idiosyncratic projects. Namely, the existance of bad banks that always herd and undertake systematic risk creates the setup in which herding with bad banks still ensures some positive bailout probability, unlike the case in which banks were homogeneous, only good banks in the economy, if all other good banks choose bank-specific projects, the bank that chose the market project would always fail with the lowest value and therefore would be first to be liquidated.

It is interesting to note that $\alpha^*_{HFL} > \alpha^*_{FL}$ where α^*_{FL} was specified in the homogeneous bank case in equation (18). This implies that the heterogeneity in bank types induces Good banks to choose more profitable idiosyncratic projects over a wider range of parameters than in the homogeneous banks' setup. Existance of banks with bad idiosyncratic projects, for which it is optimal to herd and choose the market project always, reduces herding of the good banks, banks endowed with bank-specific projects that are better than the market project.

In case when all banks were homogeneous, and bank-specific projects across all banks were more efficient than the market project, the range of parameters over which banks would choose not to herd was lower than in case when there exists heterogeneity across banks. This might imply that existance of bad banks, even

when they herd, might not be as bad for the overall banking system, from the perspective of its effect on the investment decision of the good banks.

When thinking more broadly about the interpretation of banks' optimal investment strategies, one can think of α as being the indicator of the aggregate state of the economy, since α determines the probability of high cash flows realizing for any project in this economy. Low α would indicate the economic downturn in which there are lower chances that projects would be successful, and consequently more bank failures would occur. On the other hand high α could indicate that the economy is doing well with high enough probability of high cash flows of projects being realized, therefore resulting in less banking failures.

In this context, α lower than α^*_{HFL}, as defined in Lemma 9, indicates that in economic downturn Good banks would be less willing to undertake the systematic risk and would reduce herding pressures by investing in their own idiosyncratic projects, that yield higher cash flows in the low state which in economic downturns becomes more probable to realize. On the other hand, high probability of projects yielding high cash flows could indicate that in good times, even the Good banks that have more profitable projects at disposal, are more willing to herd, as if they value less higher cash flows in the low state, given the probability of the low state is lower during economic booms. In that sense, investing in the common project increases the probability that in failure bailouts will happen, since α is too high to have enough failures of uncorrelated projects that would induce Regulator to intervene and bailout even the high value good banks, only because the probability of systemick crisis is too low, unless the banks herd.

Welfare Analysis with Heterogeneous banks

In the homogeneous case, it has already been proven that once the idiosyncratic project yields higher cash flow in the low state of the world, all else equal, that 'fail less' bailout policy induces banks to choose the investment strategy that is also ex post optimal in terms of welfare.

Once heterogeneous banks are introduced through the heterogeneity in bank-specific projects, welfare might be affected such that in some cases it may be optimal for certain banks to herd, depending on the charactereistics of thebank-specific projects.

Here, I investigate which investment strategy for good and bad banks represents

the optimal investment from the perspecitve of the total welfare, as well as, in which cases 'fail less' bailout policy actually implements the optimal outcome and remains time-consistent bailout policy, as in the homogeneous banks case. In order to do that, I calculate the total expected banking output net of expected Regulator's intervention costs, for different possible investment strategies of good and bad banks, from the perspective of date 0.

Let $E\left(\Pi\left(x_B, x_G\right)\right)$ represent the ex ante welfare, where x_B is the investment strategy of bad banks, while x_G is the investment strategy of good banks. If all banks choose to invest in the common project, then with probability α all banks have high returns and total expected banking output in that case is $n\alpha\left(\bar{R}_m + V\right)$. However, if, for example, good banks choose to invest in the bank-specific project, while bad banks still choose the market project, then with probability $Pr\left(f_G\right) = C\left(f_G, g\right)\alpha^{g-f_G}\left(1-\alpha\right)^{f_G}$, f_G of good banks failed at date 1, while the remaining $g - f_G$ obtained high returns from their bank-specific projects. Then, the expected banking output of good banks that obtained high returns will be:

$$\left(\bar{R}_G + V\right)\sum_{f_G=0}^{g}\left(g - f_G\right)Pr\left(f_G\right) = \left(\bar{R}_G + V\right)\left[g - \sum_{f_G=0}^{g} f_G Pr\left(f_G\right)\right] \quad (40)$$

since $f_G \sim B\left(g, \left(1-\alpha\right)\right)$, then the expected number of failed banks $\sum_{f_G=0}^{g} f_G Pr\left(f_G\right)$ for the Binomial distribution can be simplified to $g\left(1-\alpha\right)$. Therefore, the expression (40) representing the expected banking output of good banks that obtained high cash flows from their bank-specific projects simplifies to $\left(\bar{R}_G + V\right)g\left(1 - \left(1-\alpha\right)\right) = g\alpha\left(\bar{R}_G + V\right)$.

Since, the assumption is that all projects bank-speciifc and the market project yield equal cash flows in the high state, it follows that the expected banking output of the high cash flows will be the same regardless of the investment strategies different banks undertake. This is due to:

$n\alpha\left(\bar{R}_m + V\right) = b\alpha\left(\bar{R}_m + V\right) + g\alpha\left(\bar{R}_G + V\right) = b\alpha\left(\bar{R}_B + V\right) + g\alpha\left(\bar{R}_G + V\right) = b\alpha\left(\bar{R}_B + V\right) + g\alpha\left(\bar{R}_m + V\right)$

Thus, the expected banking output in case there are no bank failures is independent of the banks' investment strategies.

The difference in welfare depending on investment strategies will come through three components:

- the expected banking output in case low cash flows realize

- the cost of deposit insurance

- potential welfare gains of no herding $\beta_{x_B x_G}$

If all banks choose to invest in the market project, they are perfectly correlated and with probability $1 - \alpha$ all n banks will fail together, creating a systemic crisis in which the Regulator would liquidate k_m^* banks while bailout the rest. Let $E\left(\Pi\left(0,0\right)\right)$ be the total welfare when all banks invest in the common project and only undertake correlated risk.

$$
\begin{aligned}
E\left(\Pi\left(0,0\right)\right) \;=\; & n\alpha\left(\bar{R}_m + V\right) + n\left(1 - \alpha\right)\underline{R}_m \\
& - k_m^*\left(1 - \alpha\right)\left(\underline{R}_m - L\left(k_m^*\right)\right) \\
& - \left(1 - \alpha\right)\left(n\left(r - \underline{R}_m\right) - k_m^* p\left(k_m^*\right)\right)
\end{aligned}
\tag{41}
$$

If the market project yields low cash flow, all banks fail together. In that case ther Regulator will liquidate k_m^* banks through sales to outsiders. However, given that bank liquidations occur until the price outsiders pay equates the social loss from bank liquidations, when maximum number of banks is liquidated entire benefits from bank sales in terms of reduced cost of deposit insurance, will be offset by the total social loss from bank liquidations. In other words, expression (41) transforms into:

$$
\begin{aligned}
E\left(\Pi\left(0,0\right)\right) \;=\; & n\alpha\left(\bar{R}_m + V\right) + n\left(1 - \alpha\right)\underline{R}_m \\
& - \left(1 - \alpha\right)\left(n\left(r - \underline{R}_m\right)\right)
\end{aligned}
\tag{42}
$$

Namely, whenever the bank crisis is systemic, such that the Regulator liquidates the maximum number of banks, given projects in which these banks have invested, as defined in (31), entire gains from reduced cost of deposit insurance due to collected liquidity through bank sales is offset by the social loss arising from bank liquidations. Whenever more than $k_{j}*$ banks that invested in

project j has failed together, the gains and losses from bank liquidations are exactly offset. This implies that, only in cases when fewer banks fail together, i.e. when the bank crisis is not systemic will the Regulator be able to capture the additional welfare through bank liquidations. This is what I call the potential welfare gains from no herding, and for different investment decisions of banks, I will define it more precisely.

In case when good banks pick their bank-specific project, while the bad banks still invest in the market, the welfare is given as

$$
\begin{aligned}
E\left(\Pi\left(0,1\right)\right) \;=\;& b\alpha\left(\bar{R}_m + V\right) + g\alpha\left(\bar{R}_G + V\right) \\
&+\beta_{01} \\
&+b\left(1-\alpha\right)\underline{R}_m + g\left(1-\alpha\right)\underline{R}_G \\
&-b\left(1-\alpha\right)\left(r - \underline{R}_m\right) - g\left(1-\alpha\right)\left(r - \underline{R}_G\right)
\end{aligned} \tag{43}
$$

where β_{01} represents the expected gains of no herding, which in this case come from the good banks choosing the bank-specific projects which are uncorrelated. This β_{01} is defined as follows:

$$
\beta_{01} = \alpha \sum_{f_G=0}^{k_G^*} f_G Pr\left(f_G\right)\left(2L\left(f_G\right) - \underline{R}_G\right) \tag{44}
$$

When good banks invest in their idiosyncratic projects this implies that given projects are completely uncorrelated, the bank failures will also be uncorrelated. Therefore, the states in which banking crisis among the good banks is not considered systemic can occur. Whenever the Regulator is dealing with less than k_G* in case of good banks investing in projects G, he is able to capture some gains when liquidating banks through sales to outsiders. Namely, the fewer banks being sold, the price outsiders are willing to pay will be higher, and the reduction in deposit insurance cost will outweigh losses in banking output that occur due to bank liquidations. That is exactly what β_{01} is capturing.

When both good and bad banks choose to invest in their bank-specific project, and so complete lack of correlation among bank failures is achieved, the welfare is given by:

$$
\begin{aligned}
E\left(\Pi\left(1,1\right)\right) \; &= \; b\alpha\left(\bar{R}_B+V\right)+g\alpha\left(\bar{R}_G+V\right) \\
&\quad +\beta_{11} \\
&\quad +b\left(1-\alpha\right)\underline{R}_B+g\left(1-\alpha\right)\underline{R}_G \\
&\quad -b\left(1-\alpha\right)\left(r-\underline{R}_B\right)-g\left(1-\alpha\right)\left(r-\underline{R}_G\right) \qquad (45)
\end{aligned}
$$

where β_{11} captures the expected gains from no herding, i.e. bank failures being uncorrelated, and is defined as follows:

$$
\begin{aligned}
\beta_{11} \; &= \; \sum_{f_B=0}^{k_G^*}Pr\left(f_B\right)\left[\sum_{f_G=0}^{k_G^*-f_B}Pr\left(f_G\right)\left(f_B\left(2L\left(f\right)-\underline{R}_B\right)+f_G\left(2L\left(f\right)-\underline{R}_G\right)\right)\right. \tag{46} \\
&\quad +\sum_{f_G=k_G^*-f_B}^{g}Pr\left(f_G\right)\left(f_B\left(2L\left(k_G^*\right)-\underline{R}_B\right)\right)\big] \\
&\quad +\sum_{f_B=k_G^*}^{k_B^*}Pr\left(f_B\right)f_B\left(2L\left(f_B\right)-\underline{R}_B\right)
\end{aligned}
$$

Expression (46) captures three different regions in terms of number of bank failures, in which certain gains from banks undertaking idiosyncratic risk occur for the Regulator. First row of (46) captures the states in which number of bad banks' failures is below k_G^* implying that depending on the number of good banks' failures, there can be at most k_G^* bank liquidations. In terms of welfare gains, this implies that if in total there are less than k_G^* failures, both bad and good banks will be selling at a higher price than what is Regulator's minimum price he would accept for either bank, since banking crisis is not systemic and outsiders are willing to pay high enough prices for the assets they buy. If the total number of failures is greater than k_G^*, while the number of bad banks' failures is still lower than k_G^*, then Regulator will be capturing gains from bank liquidations, but only on bad banks liquidations, since for the good banks the price he receives will exactly be offseting the social losses from liquidation of the good banks. Finally, the last term of (46) represents the net gains captured from bad bank liquidations when the number of failures of bad banks is between k_G^* and k_B^*, implying that this is still region of non-systemic banking crisis when it comes to bad banks only.

When bad banks choose the bank-specific project, whereas the good banks choose the market, the total expected welfare is

$$
\begin{aligned}
E\left(\Pi\left(1,0\right)\right) \;=\; & b\alpha\left(\bar{R}_B + V\right) + g\alpha\left(\bar{R}_m + V\right) \\
& +\beta_{10} \\
& +b\left(1-\alpha\right)\underline{R}_B + g\left(1-\alpha\right)\underline{R}_m \\
& -b\left(1-\alpha\right)\left(r - \underline{R}_B\right) - g\left(1-\alpha\right)\left(r - \underline{R}_m\right) \qquad (47)
\end{aligned}
$$

where, similar to (44), gains from no herding when bad banks invest in the bank-specific project are defined as

$$
\beta_{10} = \alpha \sum_{f_B=0}^{k_B^*} f_B Pr\left(f_B\right)\left(2L\left(f_B\right) - \underline{R}_B\right) \qquad (48)
$$

Welfare given in (47) contains gains of no herding, since bad banks undertake the bank-specific project, but also contains losses from lower expected banking output when low cash flows realize, and higher cost of deposit insurance for bad banks, since project B is dominated by the market project. Whether gains outweigh the losses will differ depending on the parameters. Furthermore, given that here, I analyse welfare from the perspective of the 'fail less' bailout policy, these investment strategies would never be undertaken by banks, if banks rationally infer the bailout policy implemented. The arguments as to why the Regulator might prefer bad banks to choose the bank-specific project, while good banks herd, can be given along the lines that the Regulator might want to liquidate the bad banks, which is more easily done when their failures are uncorrelated, while always preserve the good banks in the system, which is more easily achieved when their failures are correlated, thus they are bailed out almost with certainty. However, this type of bailout policy and its broader implications are to be analysed in some future work.

In order to determine which investment strategies of banks maximize welfare, the expressions given in (42), (43) and (45) are compared. Given that the expected total banking output in case high cash flows realize is the same across different investment strategies of the banks, the Regulator that wants to maximize welfare should be implementing the bailout policy that maximizes gains from no herding

and expected banking output in the low cash flow realizations net of the costs of deposit insurance.

Since

$$E\left(\Pi\left(0,1\right)\right) - E\left(\Pi\left(0,0\right)\right) = \beta_{01} + 2 \cdot g\left(1 - \alpha\right)\left(\underline{R}_G - \underline{R}_m\right) \qquad (49)$$

it is straghtforward to see that

$$E\left(\Pi\left(0,1\right)\right) > E\left(\Pi\left(0,0\right)\right)$$

always holds. Namely, as good banks choose the bank-specific projects, while bad banks still invest in the market project, the overall welfare improves, since the herding is reduced, therefore gains from less correlated bank failures canbe captured, as well as given the good banks now choose the more efficient project their values in failure will be higher, and cost of deposit insurance lower.

Given the total welfare depends not only on how efficient are banks' investments, as observed on their own, but also what is the effect of individual banks' decisions on the overall correlation in the banking system and the frequency of systemic banking crisis occuring. It is important to determine whether no herding at all, i.e. both bad and good banks investing in their bank-specific projects might dominate, in terms of total welfare, the outcome in which bad banks herd, while good banks choose the uncorrelated idiosyncratic investment.

Therefore, I need to compare $E\left(\Pi\left(0,1\right)\right)$ and $E\left(\Pi\left(1,1\right)\right)$. Change in the bad banks' investment decision trades off the additional gains from no herding which arise from bad banks investing in their own bank-specific projects, with the losses of bad banks actually choosing less efficient projects. Given that they yield lower cash flows in the low state, bad idiosyncratic projects reduce the expected value of bad banks and also increase costs of deposit insurance relative to the case in which bad banks invest in the market project.

Consequently, if β_{11} is defined as in (??) and β_{01} is defined as in (44), then gains from bad banks not herding are specified as

$$\beta_{11} - \beta_{01} \qquad (50)$$

while the total losses from bad banks choosing less efficient B projects are

$$2b\left(1-\alpha\right)\left(\underline{R}_m - \underline{R}_B\right) \tag{51}$$

In order for the outcome with no herding at all to be preferred to the case in which only bad banks herd, gains from less frequent systemic crisis would have to outweigh losses in efficiency of the bad banks' investment projects, and the following would have to hold

$$\beta_{11} - \beta_{01} > b\left(1-\alpha\right)2\left(\underline{R}_m - \underline{R}_B\right) \tag{52}$$

In case when losses specified in (51) outweigh the expected gains from no herding, the outcome in which bad banks choose the market project would be preferred over the bad banks investing in their bank-specific projects.

This implies that there will be some threshold in terms of how bad the bank-specific project of the bad banks can actually be, in order for the regulator to still prefer bad banks to invest in it. If this project is too bad, i.e. yields too low cash flows in failure, then any gains arising from uncorrelatedness of bank failures will be outweighed by losses in efficiency since regulator has to bear higher costs of deposit insurance. Lemma 10 defines conditions under which welfare is maximized when all banks choose only idiosyncratic projects, and when it is optimal for bad banks to herd by investing in the market project.

Lemma 10   *If $\widetilde{R}_B$ represents the idiosyncratic project each bad bank is endowed with, such that $\underline{R}_B$ is the cash flow a project yields in the low state. Let $\Psi\left(\alpha, \underline{R}_G, k_j^*\right)$ and $\Omega\left(\alpha, k_j^*\right)$ be derived through algebraic manipulations of (46) and (44) when calculating $\beta_{11} - \beta_{01}$. Then, there will be some $\underline{R}_B^*$ defined as*

$$\Psi\left(\alpha, \underline{R}_G, k_j^*\right) - \Omega\left(\alpha, k_j^*\right)\underline{R}_B^* = 2b\left(1-\alpha\right)\left(\underline{R}_m - \underline{R}_B^*\right)$$

such that

- *If $\underline{R}_B > \underline{R}_B^*$, total welfare is maximized when all banks, good and bad, invest in their own bank-specific projects i.e. $E\left(\Pi\left(1,1\right)\right) > E\left(\Pi\left(0,1\right)\right)$*

- *If $\underline{R}_B \leq \underline{R}_B^*$, total welfare is maximized when bad banks invest in the market project, while the good banks invest in their bank-specific projects G, i.e. $E\left(\Pi\left(1,1\right)\right) \leq E\left(\Pi\left(0,1\right)\right)$*

Proof: See Appendix.

Lemma 10 defines the conditions under which herding might be preferred among the bad banks, given that the bank-specific projects they are endowed with are very poor relative to the market project. Therefore, in the world with heterogeneous banks, it may be that it is optimal for some banks to undertake the correlated risk, as long as the banking system also consists of banks investing in the bank-specific projects better than the market. Then, the good bank-specific projects lead to both gains from no herding and gains from investing in more efficient, higher expected value projects which so reduce the cost of deposit insurance for the Regulator.

What is important to be noted is that Regulator's ex post optimal 'fail less' bailout policy that bailouts the banks with higher value in failure, leads to the outcome that is also ex ante optimal in terms of welfare. This is when projects with which bad banks are endowed are bad enough ($\underline{R}_B \leq \underline{R}_B^*$). In addition, even in the case when bad banks idiosyncratic projects are not as bad ($\underline{R}_B > \underline{R}_B^*$), 'fail less' bailout policy still leads to better welfare outcome than what 'too many to fail' bailout policy, which does not distinguish between banks in failure would result in, and that is, herding. This is since $E\left(\Pi\left(0,1\right)\right) > E\left(\Pi\left(0,0\right)\right)$ always holds.

7   Conclusion

This paper investigates the effects of a particular regulatory channel on the banks' choice between systematic and idiosyncratic risk. Given that the regulator cares about bailing out the banks that have failed less, therefore implementing the ex post optimal 'fail less' bailout policy, this should create incentives for banks to invest in more efficient projects. This type of bailout policy resultS in the reduction of the banks' ex ante herding incentives. If bank-specific projects are better than the market project, banks choose to invest more in the bank-specific projects, therefore reducing the amount of correlated risk in the system. It is possible that some banks are endowed with poor bank-specific projects, and they would still, invest in the market, but from the welfare perspective, bad banks herding might not be purely a bad thing since it improves no herding incentives for the good banks. Overall, the amount of correlated risk in the banking system is reduced and the total welfare is improved, once the 'fail less'

bailout policy is implemented.

In this paper I compare the model in which the regulator is purely driven by the 'too many to fail' motive, with the model in which the regulator cares about saving the banks that 'failed less'. In the 'too many to fail' case, after the regulator has optimaly chosen the total number of banks that will be bailed out, any failed bank has equal probability of being saved. Therefore, in order to increase the bailout probability, banks want to ensure that the crisis is systemic enough, so they want to maximize the correlation of their failures. Thus, banks prefer to invest in the common market project which increases the probability of the systemic banking crisis and consequently, regulator's bailout intervention.

However, when banks differ in failure and the regulator cares about preserving the better banks in the system, the bailout probability of each bank will depend on its own value in failure. This induces banks to invest less in systematic risk. Although, it ensures that systemic banking crisis would occur, and regulator would intervene, it decreases bank's probability of bailout once failures have occured. For probable enough failures, low project cash flows realizing with high enough probability, banks rather choose to invest in their bank-specifc projects which might lead to uncorrelated failures, but also to higher bailout probabilities in the states in which bailouts occurs. This is due to liquidations of higher value banks being more costly for the regulator, therefore he is more willing to bailout these banks.

These results reveal that the herding incentives driven by the 'too many to fail' guarantees inherent in the regulator's bailout policy, may as well be driven in the other direction, once the regulator actually wants to save better banks.

The model also provides interesting implications when the ex-ante heterogeneity in bank-specific projects is introduced. It shows that although banks with bad bank-specific projects might still choose to herd and invest in the market, this might not be overall bad for the system, since it increases incentives of the good banks to choose their more efficient bank-specific projects.

Finally, from the welfare perspective 'fail less' bailout policy always dominates the 'too many to fail' policy by leading to the higher total welfare outcome, and is also time consistent bailout policy.

Appendix

Proof of Lemma 3 In order to find under which circumstances banks choose perfect or no interbank correlation, I need to compare $E(\pi(0))$ with $E(\pi(1))$. If $E(\pi(0)) > E(\pi(1))$ banks would choose to invest in the common project and by doing so, achieve the perfect interbank correlation, and consequently perfectly synchronized failure. Therfore, I check if the condition $E(\pi(0)) - E(\pi(1)) > 0$ holds. From equations (10) and (11) we have:

$$E(\pi(0)) - E(\pi(1)) = (1-\alpha)\underline{R}\left(\left(1 - \frac{k^*}{n}\right) - \sum_{j=k^*}^{n-1} Pr(j)\left(1 - \frac{k^*}{j+1}\right)\right) \quad (53)$$

From (53) we observe that given $j \in \{k^*, ...n-1\}$ then $1 - \frac{k^*}{j+1} \leq 1 - \frac{k^*}{n}$. Thus, the following has to hold:

$$\left(1 - \frac{k^*}{n}\right) - \sum_{j=k^*}^{n-1} Pr(j)\left(1 - \frac{k^*}{j+1}\right) \geq \left(1 - \frac{k^*}{n}\right) - \left(1 - \frac{k^*}{n}\right)\sum_{j=k^*}^{n-1} Pr(j) > 0 \quad (54)$$

Since $\sum_{j=k^*}^{n-1} Pr(j) < 1$, then inequality (54) always has to be positive. In other words, the bank will always choose to invest in the common project.

Proof of Lemma 5 Since $\alpha_{FL}^* = 1 - \left(1 - \frac{k_m^*}{n}\right)\frac{R_m}{\underline{R}_i}$ and $\alpha_b^* = \frac{k_m^*}{n}$, it follows that $\alpha_{FL}^* = 1 - (1 - \alpha_b^*)\frac{R_m}{\underline{R}_i}$. Given that both α_{FL}^* and α_b^* represent probabilities, they have to be smaller or equal than 1. I assume that both $\alpha_{FL}^* < 1$ and $\alpha_b^* < 1$ hold. Consequently, the following holds:

$$1 - \frac{R_m}{\underline{R}_i} > \alpha_b^*\left(1 - \frac{R_m}{\underline{R}_i}\right)$$

Stated differently

$$1 - \frac{R_m}{\underline{R}_i} + \alpha_b^*\frac{R_m}{\underline{R}_i} > \alpha_b^* \quad (55)$$

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From definition of α^*_{FL} inequality (55), it is clear that $\alpha^*_b < \alpha^*_{FL}$ has to hold.

Derivation of $E\left(\Pi\left(1\right)\right)$ Therefore (23) can be transformed into:

$$
\begin{aligned}
E\left(\Pi\left(1\right)\right) \;=\;& n\alpha\left(\overline{R}_i + V\right) + \\
& + \sum_{j=0}^{k_i^*-1} jPr\left(j\right) \cdot L\left(j\right) - \sum_{j=0}^{k_i^*-1} jPr\left(j\right)\left(r - \underline{R}_i - p\left(j\right)\right) \\
& + \underline{R}_i \sum_{j=k_i^*}^{n} jPr\left(j\right) - k_i^*\left(\underline{R}_i - L\left(k_i^*\right)\right)\sum_{j=k_i^*}^{n} Pr\left(j\right) \qquad (56) \\
& - \sum_{j=k_i^*}^{n} Pr\left(j\right)\left(j\left(r - \underline{R}_i\right) - k_i^* \cdot p\left(k_i^*\right)\right)
\end{aligned}
$$

In equilibrium, in the states in which more than k_i^* banks fail together, the Regulator liquidates maximum number of banks such that $\underline{R}_i - L\left(k_i^*\right) = p\left(k_i^*\right)$ holds. This implies that the gains from bank sales in terms of collected liquidity are entirely offset by the social loss arising from buyers being the inefficient users of the banking assets. Then (56) can be further simplified to the following:

$$
\begin{aligned}
E\left(\Pi\left(1\right)\right) \;=\;& n\alpha\left(\overline{R}_i + V\right) + \\
& + \sum_{j=0}^{k_i^*} jPr\left(j\right) \cdot L\left(j\right) + \underline{R}_i \cdot \sum_{j=k_i^*+1}^{n} jPr\left(j\right) \qquad (57) \\
& - \left(1 - \alpha\right) n\left(r - \underline{R}_i\right) + \sum_{j=0}^{k_i^*} jPr\left(j\right) \cdot p\left(j\right)
\end{aligned}
$$

where again the formula for expected value of binomially distributed variable has been used. In order to obtain the expression (57), I used the following simplification:

$$
\left(r - \underline{R}_i\right)\left(\sum_{j=0}^{k_i^*-1} jPr\left(j\right) + \sum_{j=k_i^*}^{n} jPr\left(j\right)\right) = \left(r - \underline{R}_i\right)\sum_{j=0}^{n} jPr\left(j\right) = \left(r - \underline{R}_i\right)\left(1 - \alpha\right) n
$$

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Given that the participation constraint of the outsiders is binding $p(j) = L(j)$ for every number of liquidated banks j, and that $\sum_{j=k_i^*+1}^{n} jPr(j) = \sum_{j=0}^{n} jPr(j) - \sum_{j=0}^{k_i^*} jPr(j)$, expression (25) follows.

Proof of Lemma 8 In order to determine the optimal investment strategy of Bad banks, given that payoffs from both investment projects are the same and independent of other banks' choice in case when bank does not fail, I will compare the expected bailout subsidies of the two investment projects, given the Good banks investment strategy.

First, I prove that it is optimal for the bad banks to choose the market project when the good banks are choosing their own idiosyncratic project G.

If $x_B = 1$ the bad banks' bailout probability, regardless of what happens with the good banks is the following:

$$Pr(f_B > k_B^*)Pr\left(Bailout \mid f, \tilde{R}_B\right) = \sum_{f_B=k_B^*}^{b-1} Pr(f_B)\left(1 - \frac{k_B^*}{f_B+1}\right) \quad (58)$$

here the total number of bad banks that failed is $f_B + 1$ since f_B represents the number of bad banks that failed beside the representative Bad bank failing.

From (58) it can be concluded that the state with the maximum bailout probability is the state in which total number of failures is $f_B + 1 = b$, therefore maximum possible bailout probability is:

$$1 - \frac{k_B^*}{b} \geq 1 - \frac{k_B^*}{f_B+1}$$

for any f_B. Then, the following is true:

$$\sum_{f_B=k_B^*}^{b-1} Pr(f_B)\left(1 - \frac{k_B^*}{f_B+1}\right) < \left(1 - \frac{k_B^*}{b}\right)\sum_{f_B=k_B^*}^{b-1} Pr(f_B) \quad (59)$$

Since f_B represents the total number of failures among bad banks, this is a binomially distributed variable $f_B \sim B(b, 1-\alpha)$, and therefore since $\sum_{f_B=k_B^*}^{b-1}$

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$Pr(f_B) < 1$, inequality (59) can be further simplified to:

$$\sum_{f_B=k_B^*}^{b-1} Pr(f_B) \left(1 - \frac{k_B^*}{f_B+1}\right) < 1 - \frac{k_B^*}{b} < 1 - \frac{k_m^*}{b} \tag{60}$$

since $k_m^* < k_B^*$ holds.

From the last inequality itk follows that $E(\pi_B(0,1)) > E(\pi_B(1,1))$ and therefore, investing in the market project is bad banks' best response when the good banks are investing in their idiosyncratic project G. In order to prove that this is indeed the bad banks best response, even when the good banks are choosing the market project, I have to show that

$$1 - \frac{k_B^*}{b} < 1 - \frac{k_m^*}{n} \tag{61}$$

holds as well. Then, it would be that $E(\pi_B(0,0)) > E(\pi_B(1,0))$ and indeed investing in the market project would be bad banks' dominant strategy. Since $k_m^* < k_B^*$ holds and $b < n$ the condition (61) holds, therefore investing in the market project is dominant strategy for the bad banks.

Proof of Lemma 9   In order to be optimal for Good banks to invest in their idiosyncratic projects, while Bad banks are investing in the market project, it would have to be:

$$E(\pi_G(0,1)) - E(\pi_G(0,0)) > 0$$

Given equations defined in (38) and (39), this translates into:

$$\alpha \sum_{f_G=k_G^*}^{g-1} Pr(f_G) \left(1 - \frac{k_G^*}{f_G+1}\right) + (1-\alpha) \cdot 1 > \left(1 - \frac{k_m^*}{n}\right)\frac{R_m}{\underline{R}_G} \tag{62}$$

This can further be written as:

$$\alpha(1 - \Theta(\alpha)) > 1 - \left(1 - \frac{k_m^*}{n}\right)\frac{R_m}{\underline{R}_G} \tag{63}$$

51

where $\Theta(\alpha) = \sum_{f_G=k_G^*}^{g-1} Pr(f_G)\left(1 - \frac{k_G^*}{f_G+1}\right)$, and $Pr(f_G) = C(f_G, g)\alpha^{g-f_G}(1-\alpha)^{f_G}$.

There will be some α_{HFL}^*, implicitly defined as:

$$\alpha_{HFL}^*(1 - \Theta(\alpha_{HFL}^*)) = 1 - \left(1 - \frac{k_m^*}{n}\right)\frac{R_m}{\underline{R}_G}$$

Then, since $\Theta(\alpha)$ is a decreasing function in α, the result of Lemma 9 follows:

$$E(\pi_G(0,1)) - E(\pi_G(0,0)) > 0$$

when

$$\alpha < \alpha_{HFL}^*$$

Proof of Lemma 10   If $\beta_{11} - \beta_{01} > b(1-\alpha)2(\underline{R}_m - \underline{R}_B)$ holds, then the welfare is maximized when all banks invest in their own bank-specific projects. Using definitions of β_{11} as in (??) and β_{01} as in (44), it is possible to calculate the lowest payoff of the bad banks' idiosyncratic project $\underline{R}_B^*$ for which it would be optimal for bad banks to choose their idiosyncratic project, although it is dominated by the market project $(\underline{R}_m > \underline{R}_B)$.

$$\beta_{11} - \beta_{01} = 2b(1-\alpha)(\underline{R}_m - \underline{R}_B^*)$$

Then, by replacing β_{11} and β_{01} with their definitions

$$\sum_{f_B=0}^{k_G^*} Pr(f_B)\,[\sum_{f_G=0}^{k_G^*-f_B} Pr(f_G)\,(f_B(2L(f) - \underline{R}_B^*) + f_G(2L(f) - \underline{R}_G))$$

$$+ \sum_{f_G=k_G^*-f_B}^{g} Pr(f_G)\,(f_B(2L(k_G^*) - \underline{R}_B^*))]$$

$$+ \sum_{f_B=k_G^*}^{k_B^*} Pr(f_B)f_B(2L(f_B) - \underline{R}_B^*)$$

$$-\alpha\sum_{f_G=0}^{k_G^*} f_G Pr(f_G)(2L(f_G) - \underline{R}_G) \quad = \quad 2b(1-\alpha)(\underline{R}_m - \underline{R}_B^*)$$

This can be further transformed into

$$
\sum_{f_B=0}^{k_G^*} Pr(f_B) \left[\sum_{f_G=0}^{k_G^*-f_B} Pr(f_G) f \cdot 2L(f) + \sum_{f_G=k_G^*-f_B}^{g} Pr(f_G) f_B \cdot 2L(k_G^*) \right] +
$$

$$
\sum_{f_B=k_G^*}^{k_B^*} Pr(f_B) f_B \cdot 2L(f_B) - \alpha \sum_{f_G=0}^{k_G^*} f_G Pr(f_G)(2L(f_G) - \underline{R}_G)
$$

$$
- \left[\sum_{f_B=0}^{k_G^*} Pr(f_B) f_B \left(\sum_{f_G=0}^{g} Pr(f_G) \right) + \sum_{f_B=k_G^*}^{k_B^*} Pr(f_B) f_B \right] \underline{R}_B^* \quad = \quad 2b(1-\alpha)(\underline{R}_m - \underline{R}_B^*) \quad (64)
$$

Then, I define $\Psi\left(\alpha, \underline{R}_G, k_j^*\right)$ and $\Omega\left(\alpha, k_j^*\right)$, as follows

$$
\Psi\left(\alpha, \underline{R}_G, k_j^*\right) \quad = \quad \sum_{f_B=0}^{k_G^*} Pr(f_B) \left[\sum_{f_G=0}^{k_G^*-f_B} Pr(f_G) f \cdot 2L(f) + \sum_{f_G=k_G^*-f_B}^{g} Pr(f_G) f_B \cdot 2L(k_G^*) \right] \quad (65)
$$

$$
+ \quad \sum_{f_B=k_G^*}^{k_B^*} Pr(f_B) f_B \cdot 2L(f_B) - \alpha \sum_{f_G=0}^{k_G^*} f_G Pr(f_G)(2L(f_G) - \underline{R}_G)
$$

$$
\Omega\left(\alpha, k_j^*\right) = \left[\sum_{f_B=0}^{k_G^*} Pr(f_B) f_B \left(\sum_{f_G=0}^{g} Pr(f_G) \right) + \sum_{f_B=k_G^*}^{k_B^*} Pr(f_B) f_B \right] \quad (66)
$$

Then equality (64) can be written as

$$
\Psi\left(\alpha, \underline{R}_G, k_j^*\right) - \Omega\left(\alpha, k_j^*\right) \underline{R}_B^* = 2b(1-\alpha)(\underline{R}_m - \underline{R}_B^*) \quad (67)
$$

Finally, the lowest cash flow that the bad idiosyncratic projects can yield, while still being welfare maximizing for the bad banks to undertake these idiosyncratic projects is given as

$$
\underline{R}_B^* = \frac{\Psi\left(\alpha, \underline{R}_G, k_j^*\right) - 2b(1-\alpha)\underline{R}_m}{\Omega\left(\alpha, k_j^*\right) - 2b(1-\alpha)} \quad (68)
$$

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